Set 3: Informed Heuristic Search

ICS 271 Fall 2014

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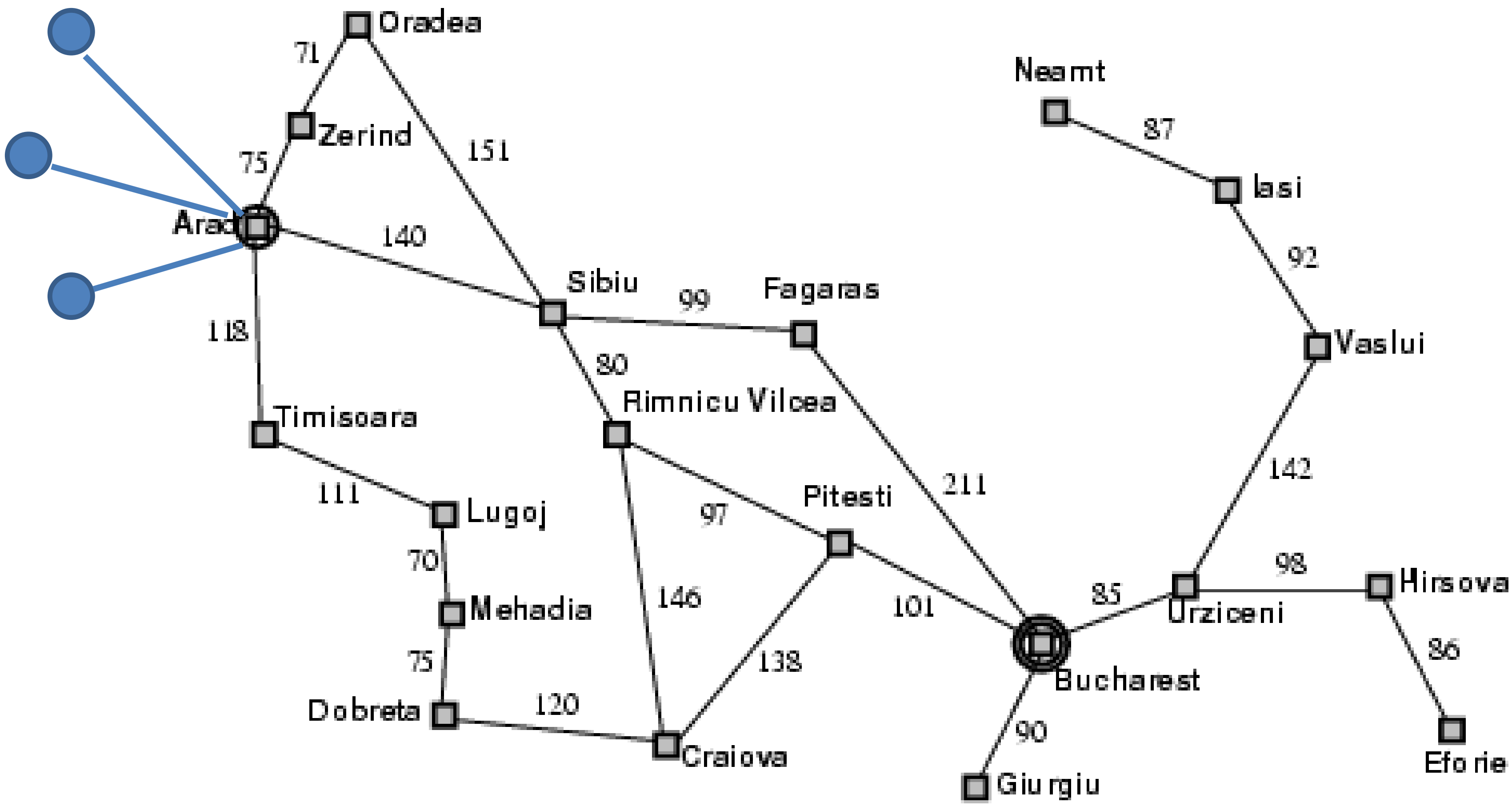
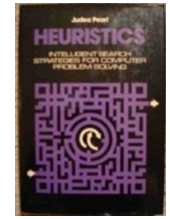
Overview

- Heuristics and Optimal search strategies
  - heuristics
  - hill-climbing algorithms
  - Best-First search
  - A*: optimal search using heuristics
  - Properties of A*
    - admissibility,
    - consistency,
    - accuracy and dominance
    - Optimal efficiency of A*
  - Branch and Bound
  - Iterative deepening A*
  - Automatic generation of heuristics

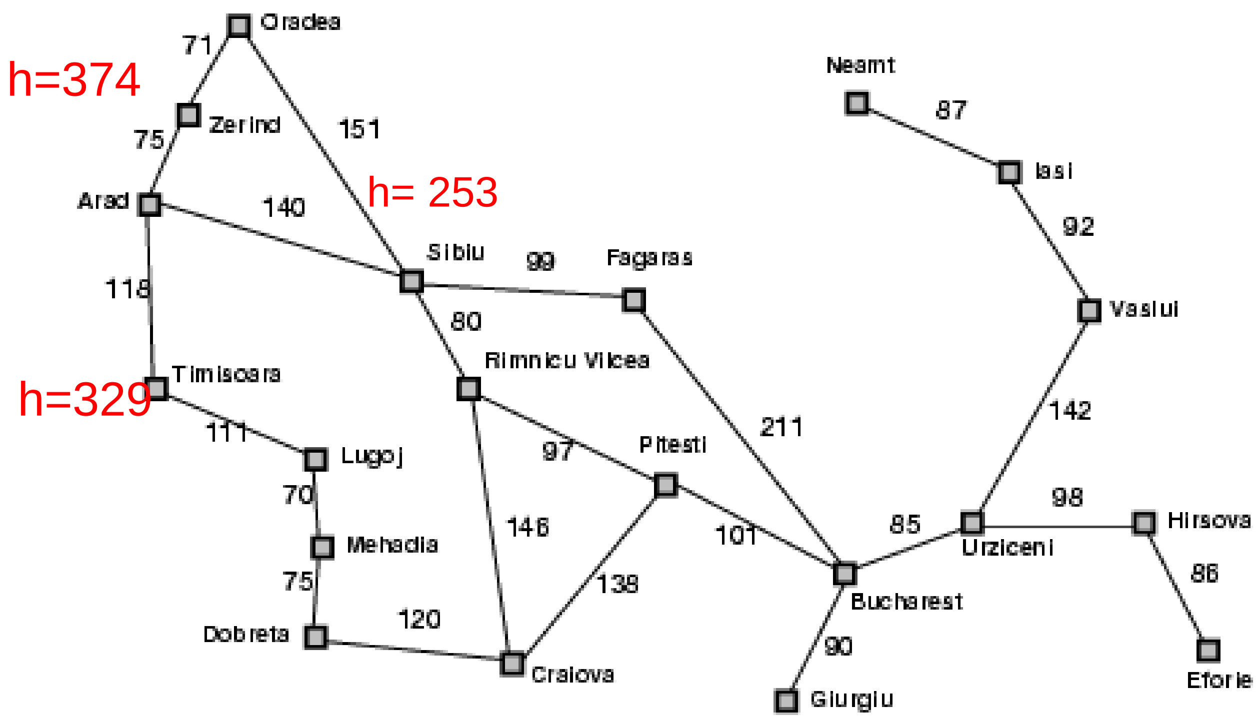
What is a heuristic?



Heuristic Search



- State-Space Search: every problem is like search of a map
- A problem solving robot finds a path in a state-space graph from start state to goal state, using heuristics



Straight–line distance to Bucharest

Arad	366
Bucharest	0
Craiova	160
Dobreta	242
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Fagaras	176
Giurgiu	77
Hirsova	151
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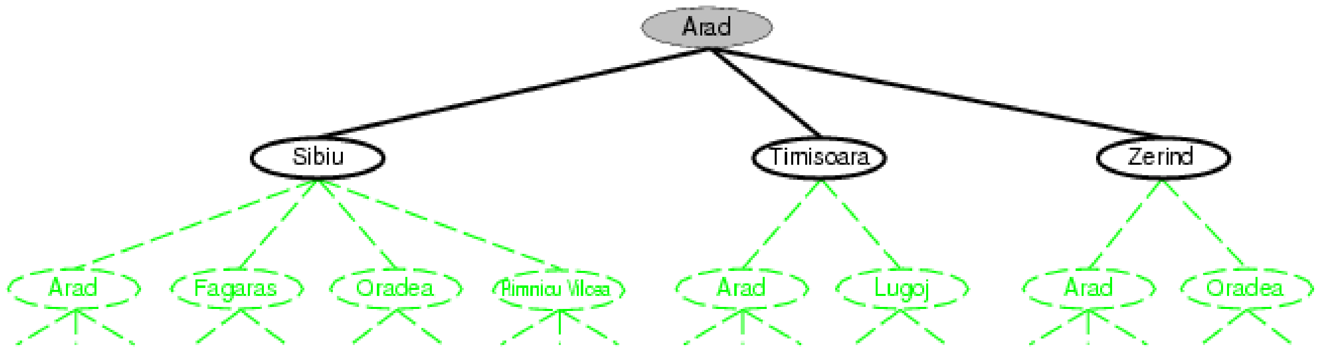
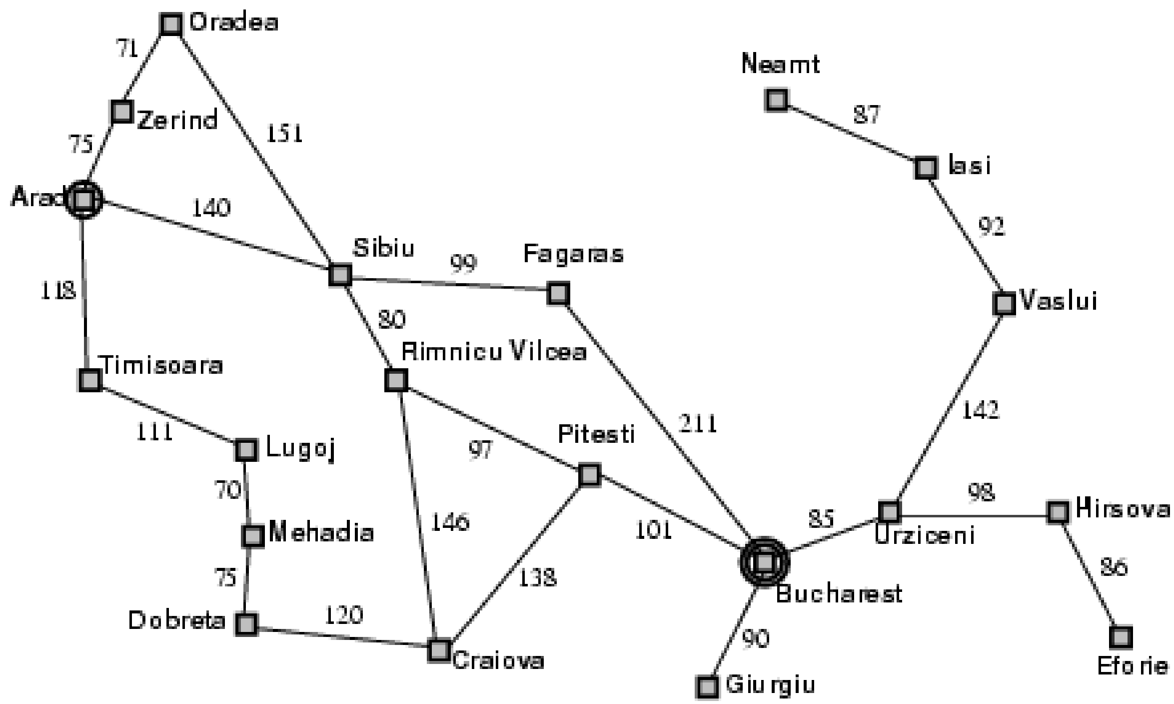
Heuristic = straight-line distance

State Space for Path Finding in a Map





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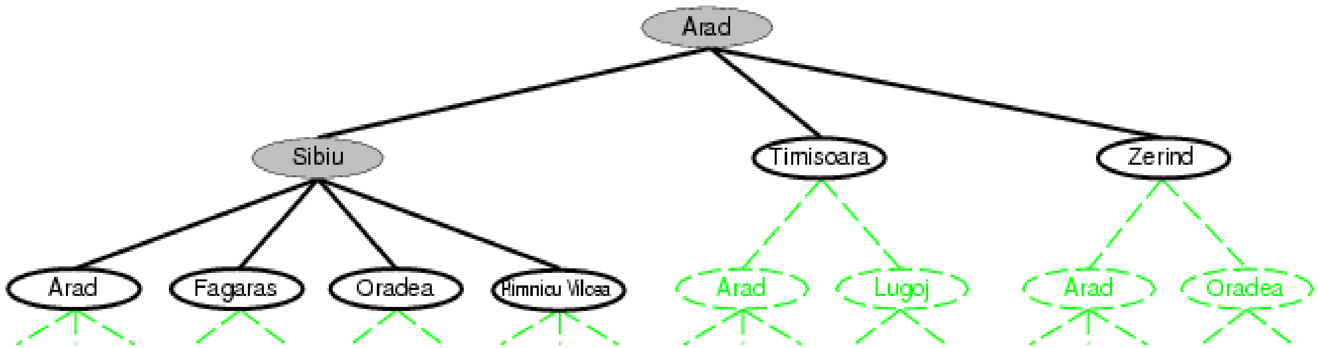
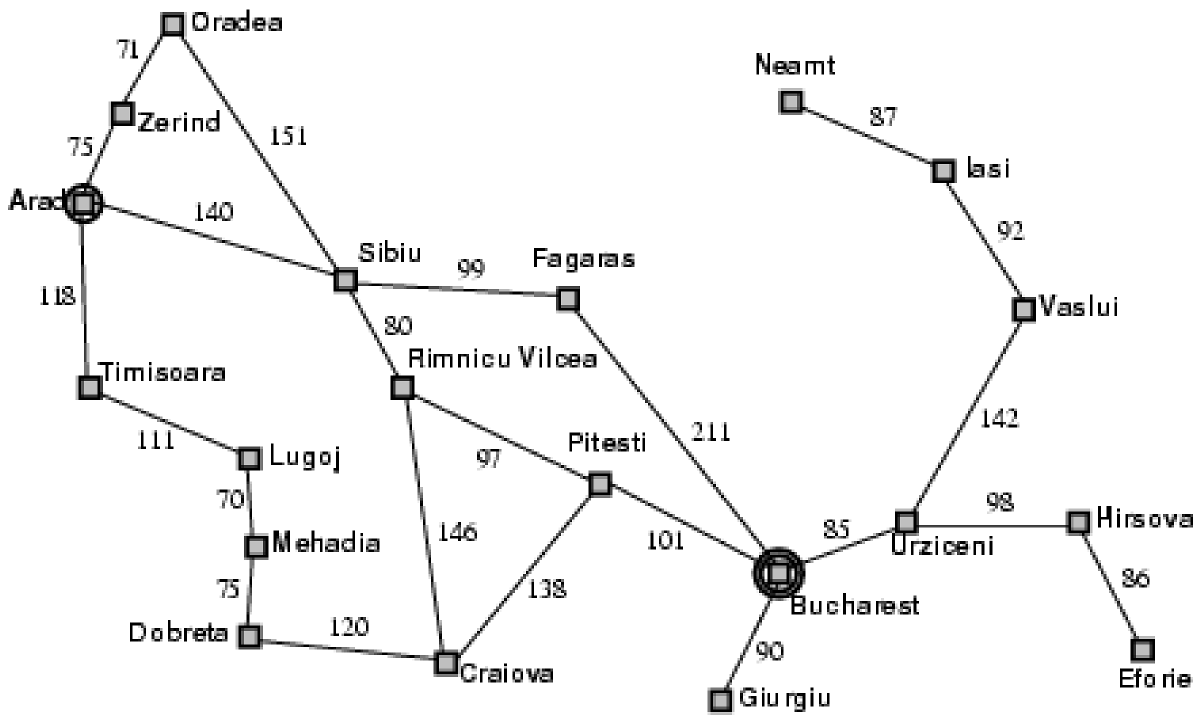
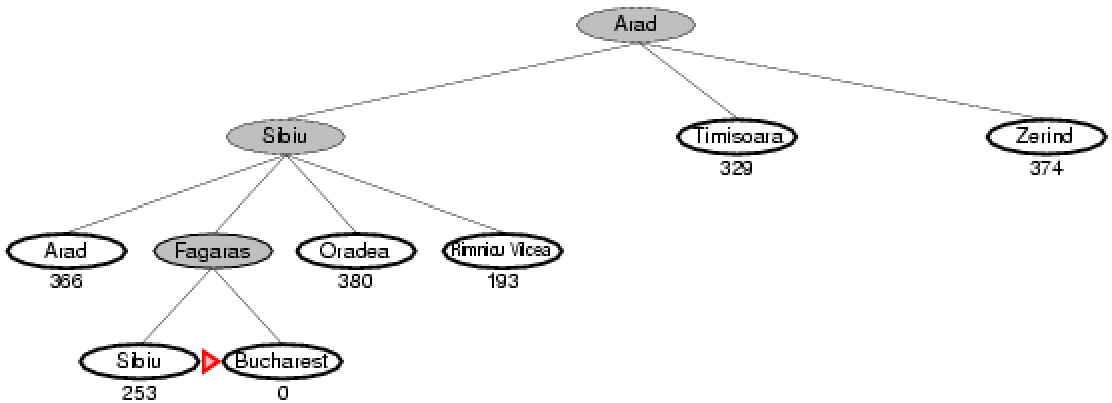
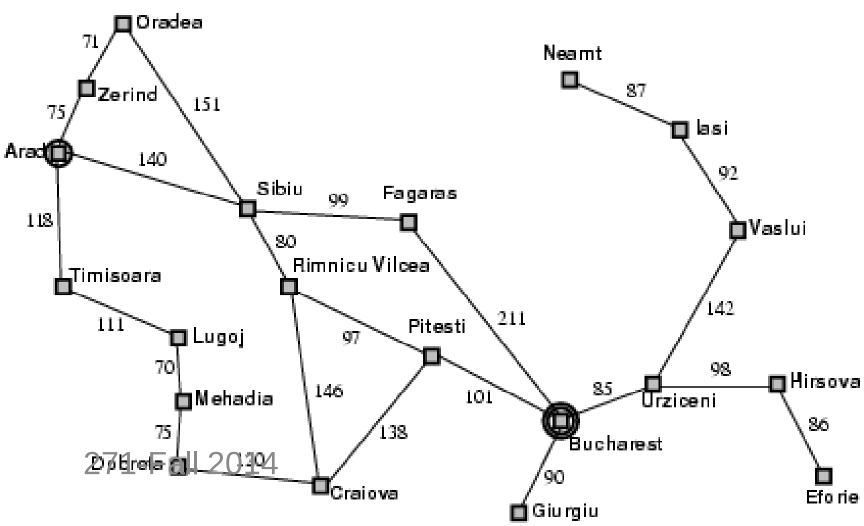




Greedy Search Example





State Space of the 8 Puzzle Problem

Initial state

goal

1	2	3
4	5	6
7	8	

8-puzzle: 181,440 states
15-puzzle: 1.3 trilion
24-puzzle: 10^25

Search space exponential

Use Heuristics as people do

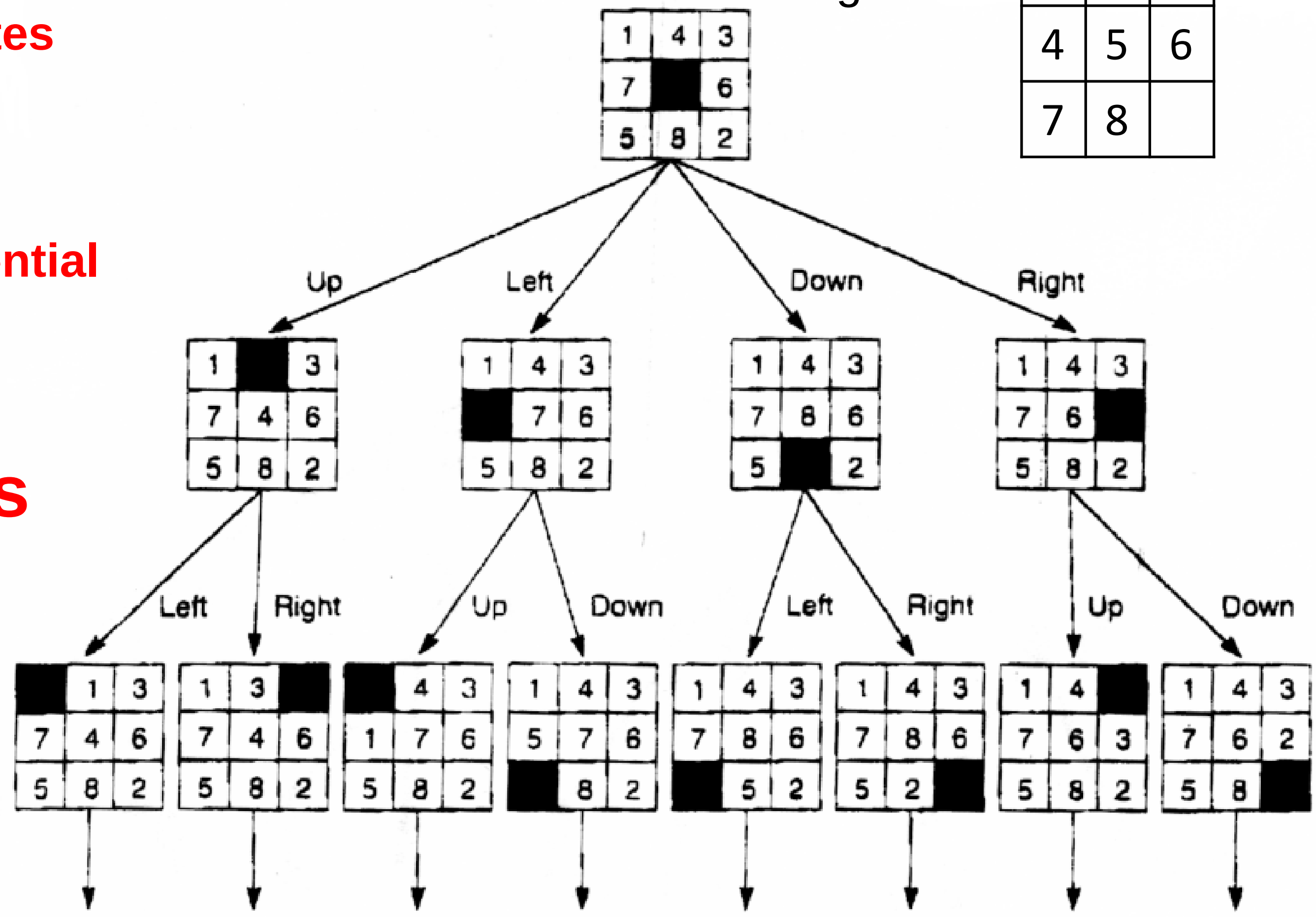


Figure 3.6 State space of the 8-puzzle generated by "move blank" operations.

State Space of the 8 Puzzle Problem

1	2	3
4	5	6
7	8	

h1 = number of misplaced tiles

h2 = Manhattan distance

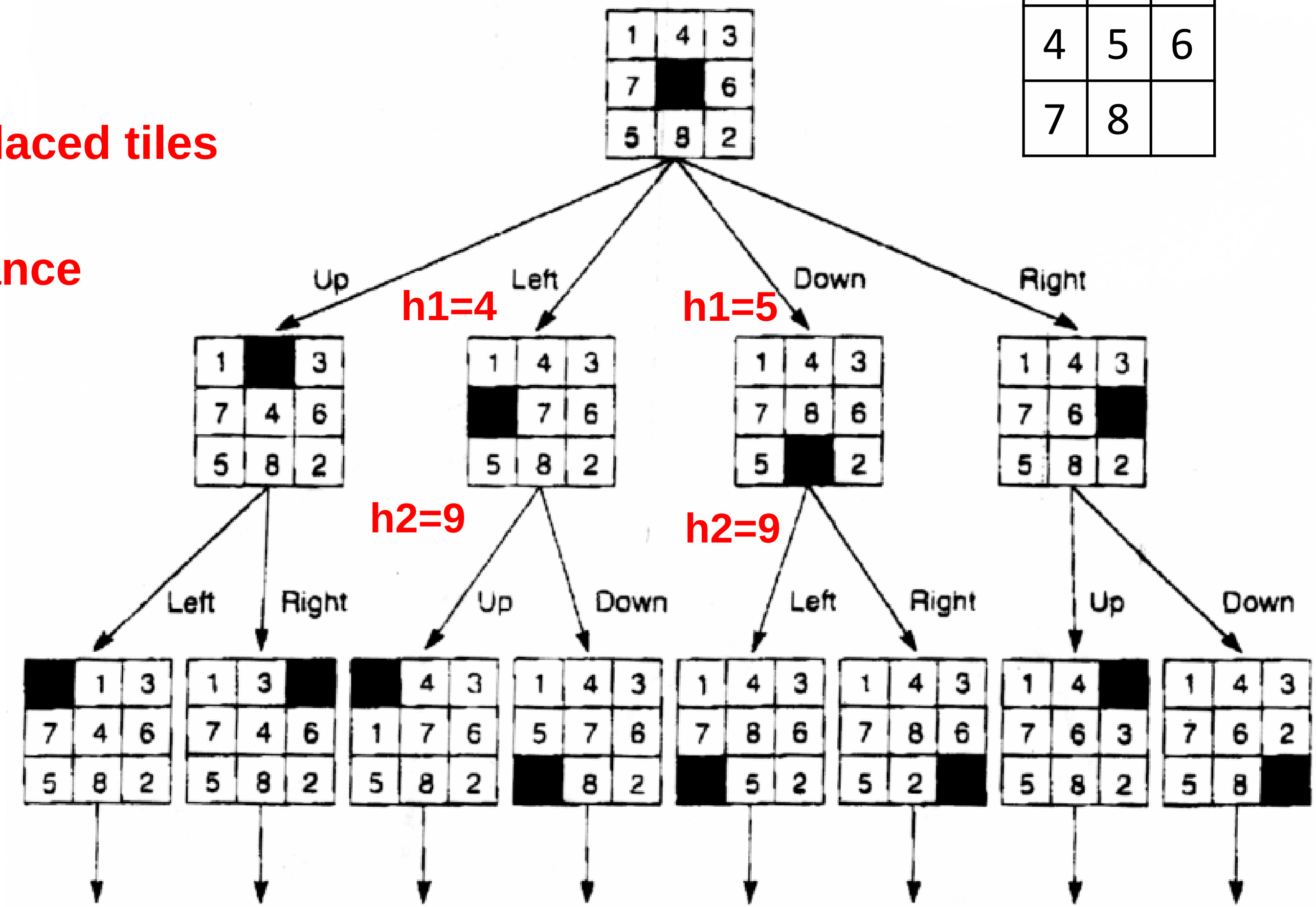
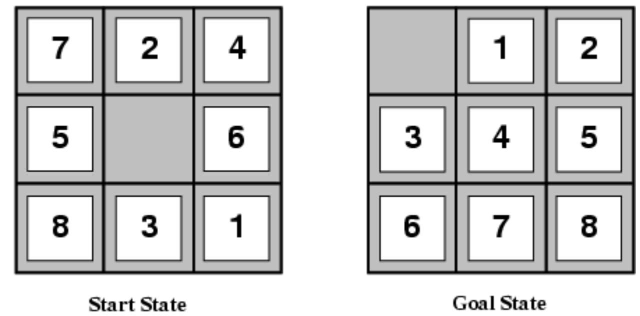


Figure 3.6 State space of the 8-puzzle generated by "move blank" operations.

What are Heuristics

- **Rule of thumb, intuition**
- **A quick way to estimate how close we are to the goal. How close is a state to the goal..**
- Pearl: “the ever-amazing observation of how much people can accomplish with that simplistic, unreliable information source known as *intuition*.”

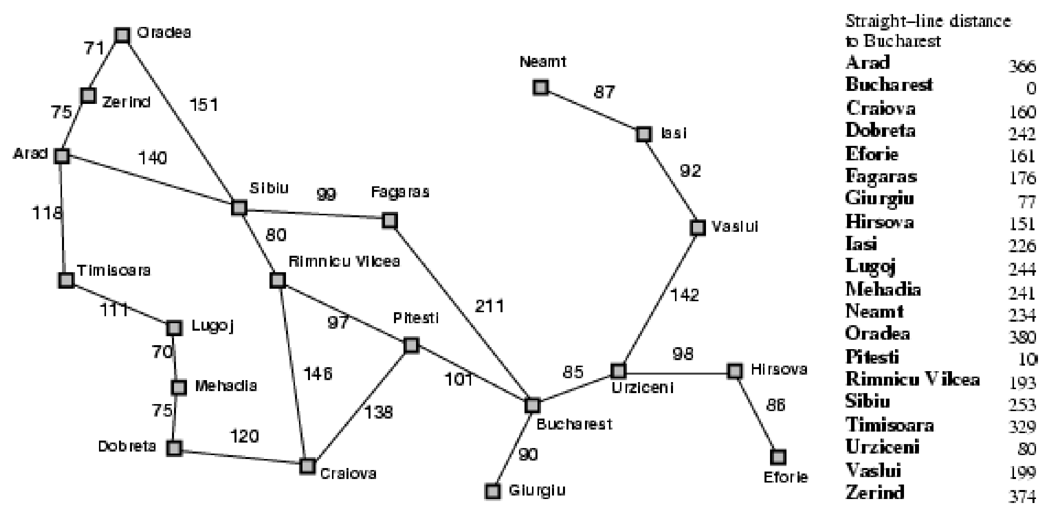


8-puzzle

- **h1(n): number of misplaced tiles**
- **h2(n): Manhattan distance**

$h_1(S)$ = ? 8
$h_2(S)$ = ? 3+1+2+2+2+3+3+2 = 18

- Path-finding on a map
  - Euclidean distance

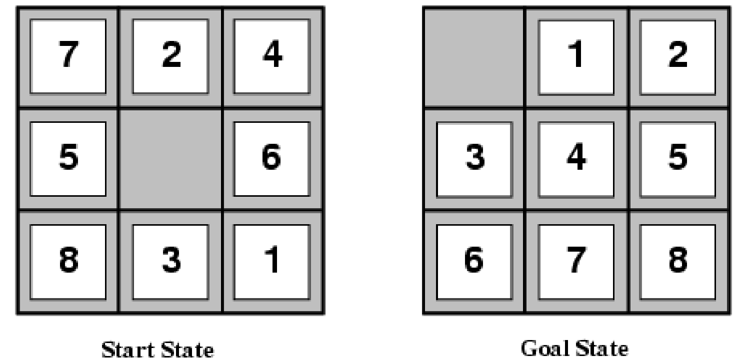


Problem: Finding a Minimum Cost Path

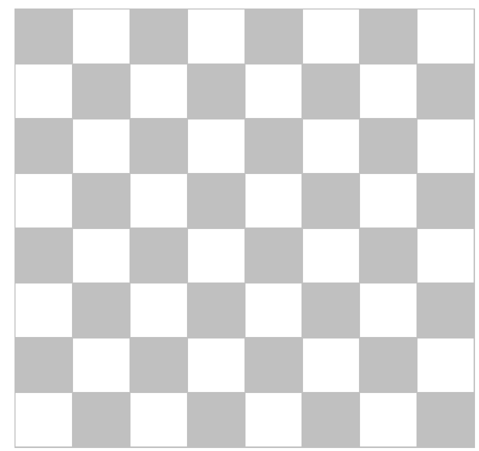
- Previously we wanted an path with minimum number of steps. Now, we want the minimum cost path to a goal G
  - Cost of a path = sum of individual transitions along path
- Examples of path-cost:
  - Navigation
    - path-cost = distance to node in miles
      - minimum => minimum time, least fuel
  - VLSI Design
    - path-cost = length of wires between chips
      - minimum => least clock/signal delay
  - 8-Puzzle
    - path-cost = number of pieces moved
      - minimum => least time to solve the puzzle
- Algorithm: Uniform-cost search… still somewhat blind

Heuristic Functions

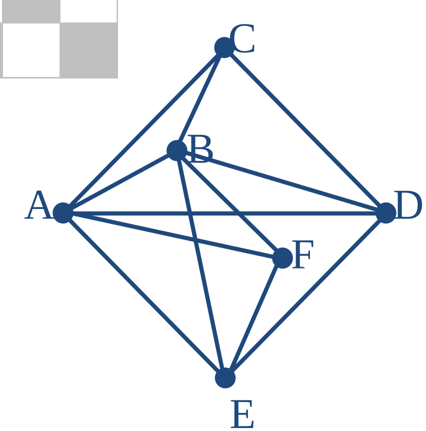
- 8-puzzle
  - Number of misplaced tiles
  - Manhattan distance
  - Gaschnig's



- 8-queen
  - Number of future feasible slots
  - Min number of feasible slots in a row
  - Min number of conflicts (in complete assignments states)



- Travelling salesperson
  - Minimum spanning tree
  - Minimum assignment problem



Best-First (Greedy) Search:
f(n) = number of misplaced tiles

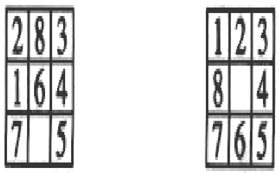
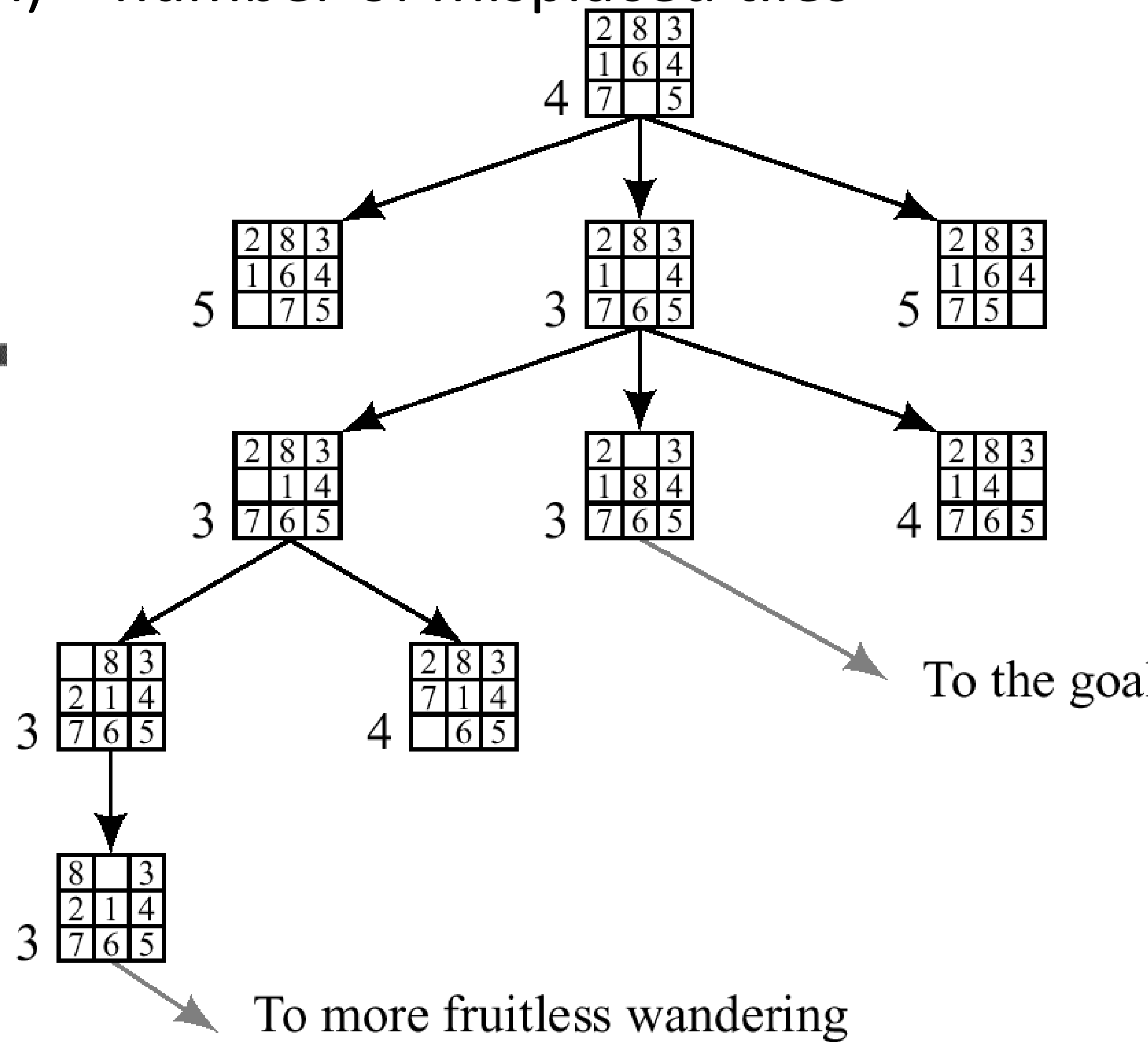
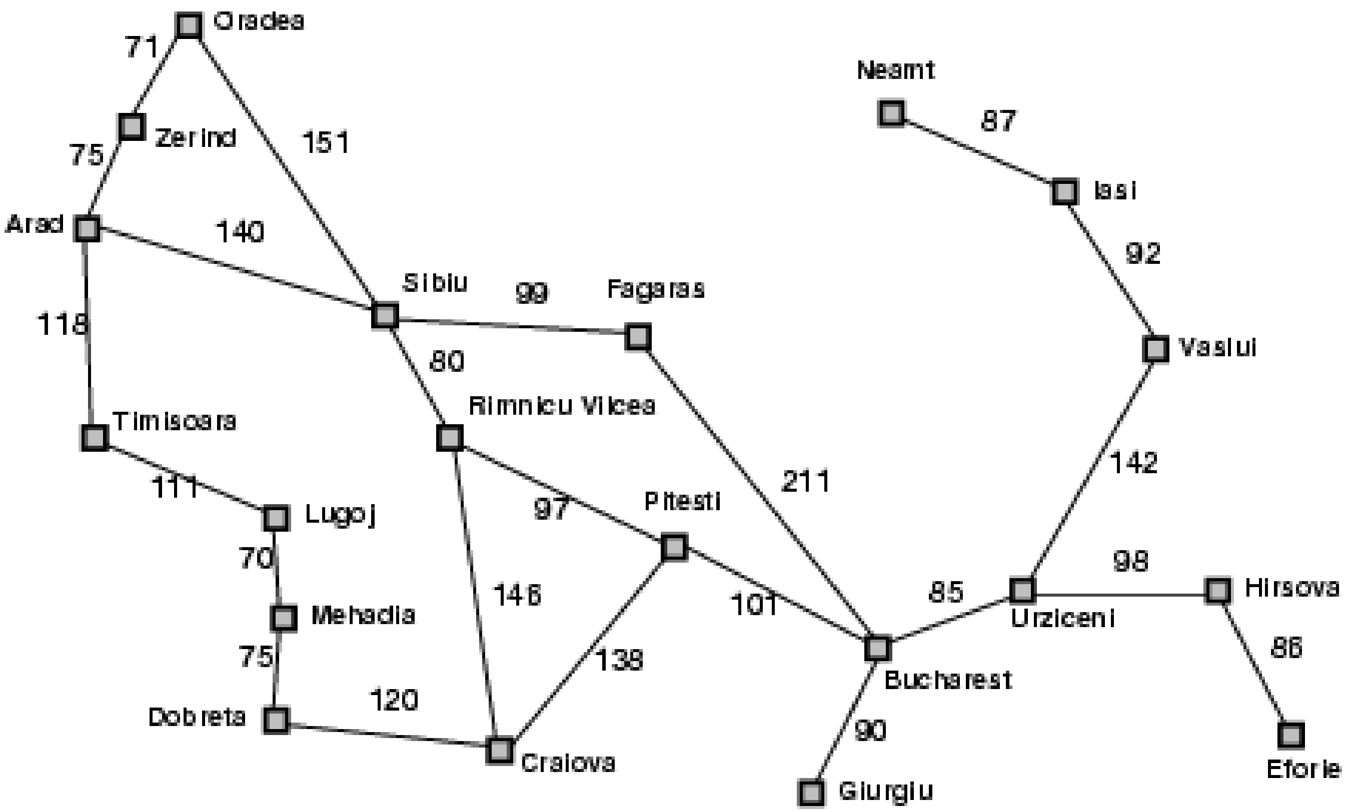


Figure 8.1

Start and Goal Configurations for the Eight-Puzzle



Romania with Step Costs in km



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Greedy Best-First Search

- Evaluation function $f(n) = h(n)$ (heuristic)

  = estimate of cost from n to *goal*

- e.g., $h_{SLD}(n)$ = straight-line distance from n to Bucharest

- Greedy best-first search expands the node that appears to be closest to goal

Greedy Best-First Search Example

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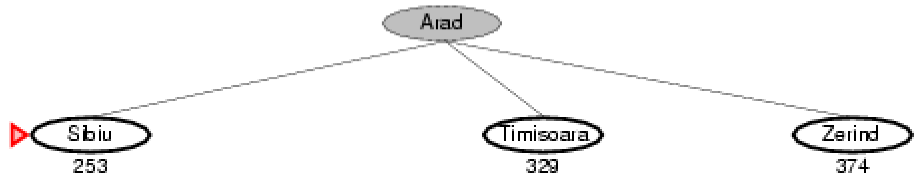

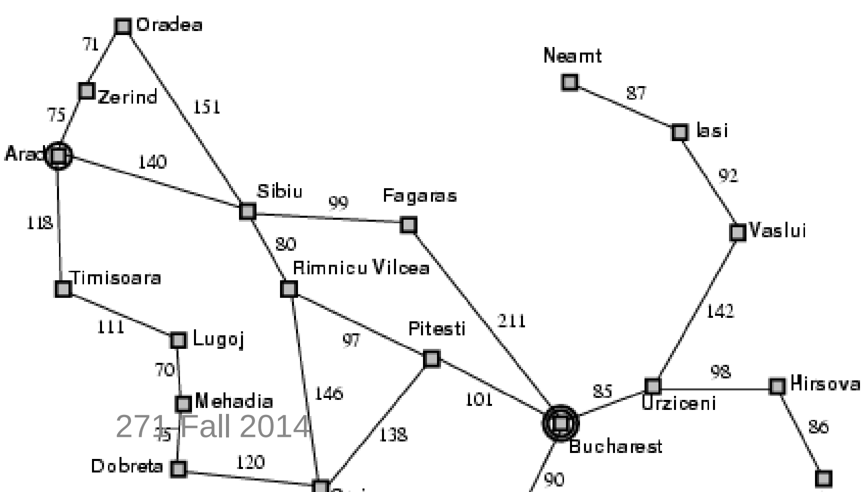

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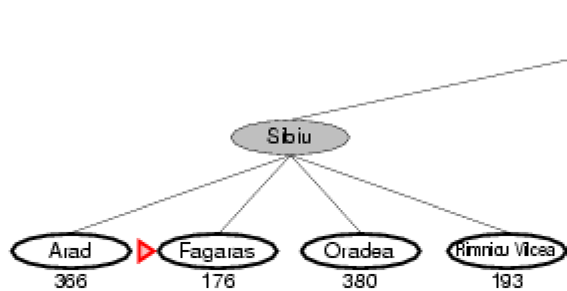

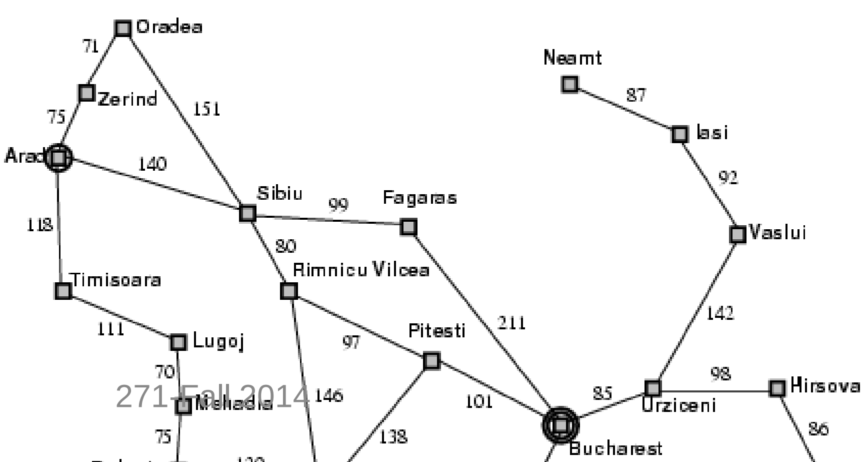

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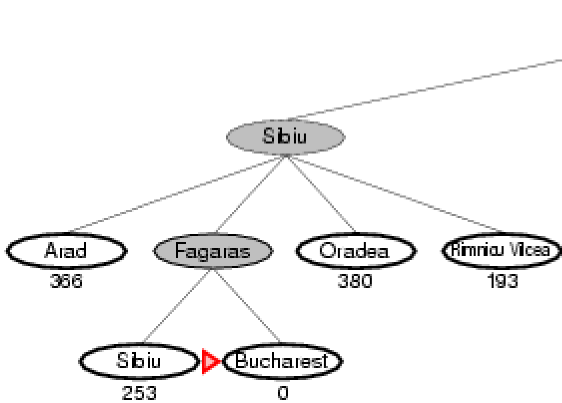

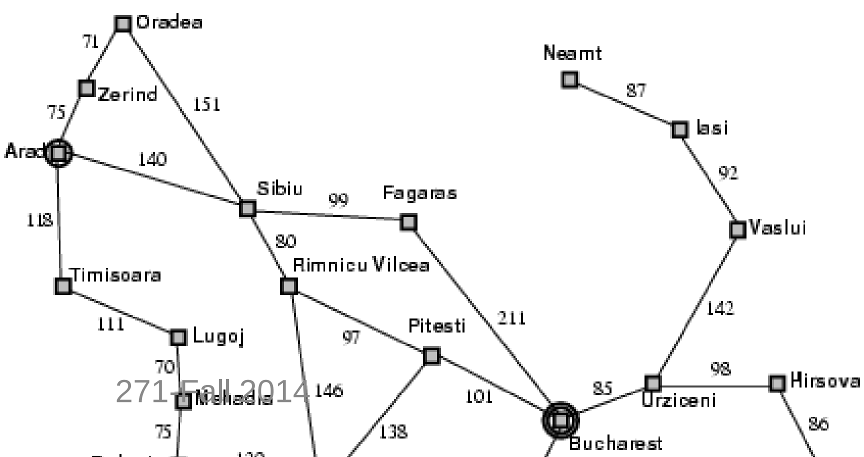

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Problems with Greedy Search

- Not complete
  - Get stuck on local minimas and plateaus
- Irrevocable
- Not optimal
- Infinite loops
- Can we incorporate heuristics in systematic search?

Informed Search - Heuristic Search

- How to use heuristic knowledge in systematic search?
- Where ? (in node expansion? hill-climbing ?)
- Best-first:
  - select the best from **all** the nodes encountered so far in OPEN.
  - "good" use heuristics
- Heuristic estimates value of a node
  - promise of a node
  - difficulty of solving the subproblem
  - quality of solution represented by node
  - the amount of information gained.
- f(n)- heuristic evaluation function.
  - depends on n, goal, search so far, domain

A^* Search

- Idea:
  - avoid expanding paths that are already expensive
  - focus on paths that show promise

- Evaluation function $f(n) = g(n) + h(n)$

- $g(n)$ = cost so far to reach n

- $h(n)$ = estimated cost from n to goal

- $f(n)$ = estimated total cost of path through n to goal

Best-First Algorithm *BF (*)*

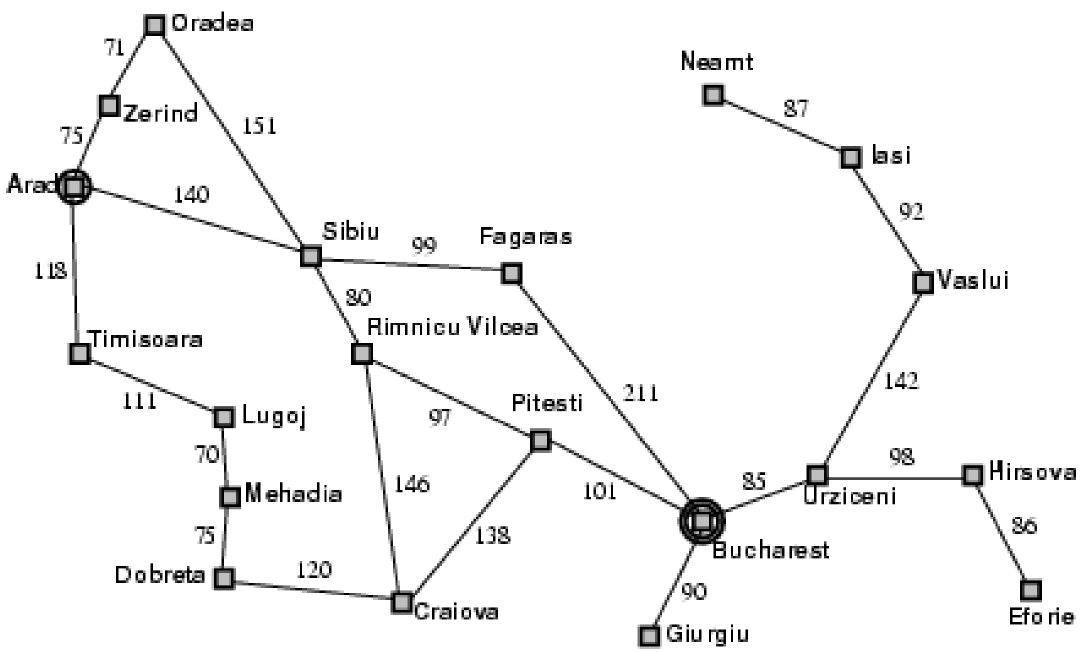
1. Put the start node *s* on a list called *OPEN* of unexpanded nodes.
2. If *OPEN* is empty exit with failure; no solutions exists.
3. Remove the first *OPEN* node *n* at which f is minimum (break ties arbitrarily), and place it on a list called *CLOSED* to be used for expanded nodes.
4. Expand node *n*, generating all it's successors with pointers back to *n*.
5. If any of *n*'s successors is a goal node, exit successfully with the solution obtained by tracing the path along the pointers from the goal back to *s*.

6. For every successor *n'* on *n*:
   a. Calculate *f* (*n'*).
   b. if *n'* was neither on *OPEN* nor on *CLOSED*, add it to *OPEN*. Attach a pointer from *n'* back to *n*. Assign the newly computed *f(n')* to node *n'*.
   c. if *n'* already resided on *OPEN* or *CLOSED*, compare the newly computed *f(n')* with the value previously assigned to *n'*. If the old value is lower, discard the newly generated node. If the new value is lower, substitute it for the old (*n'* now points back to *n* instead of to its previous predecessor). If the matching node *n'* resided on *CLOSED*, move it back to *OPEN*.

7. Go to step 2.

* With tests for duplicate nodes.

A* Search Example



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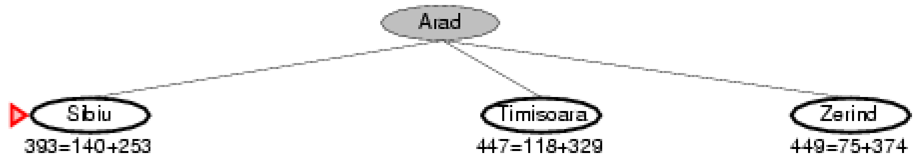

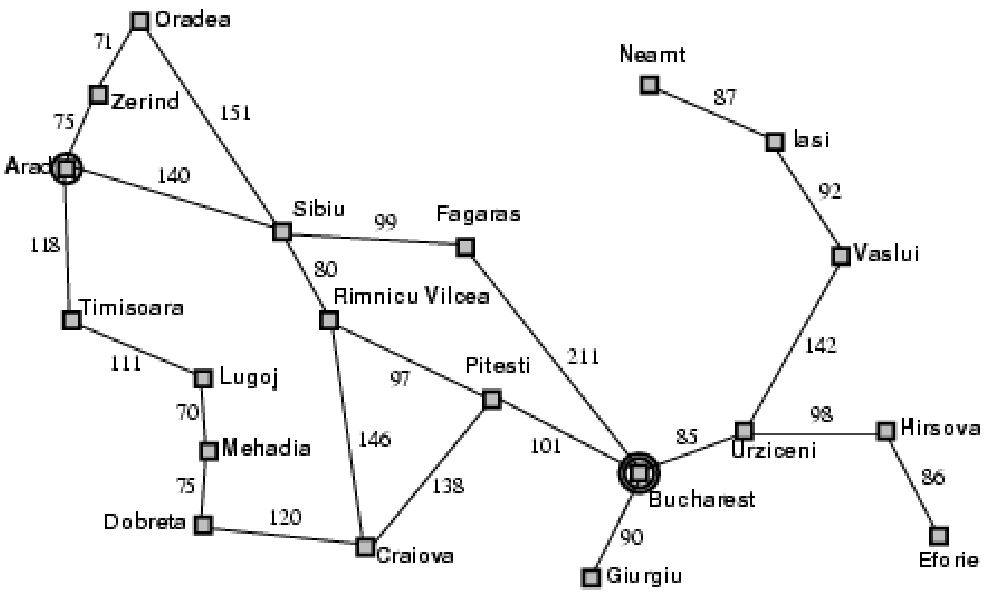

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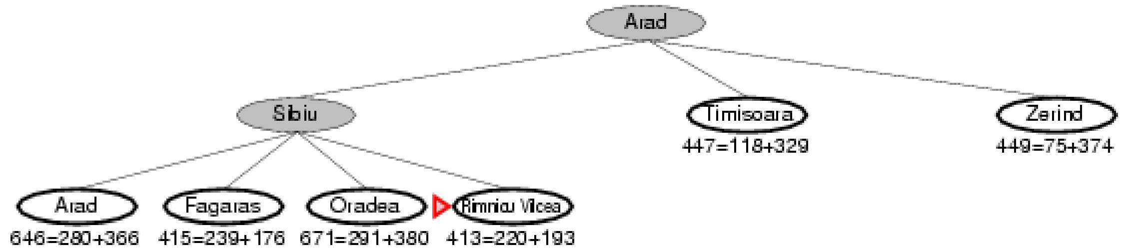

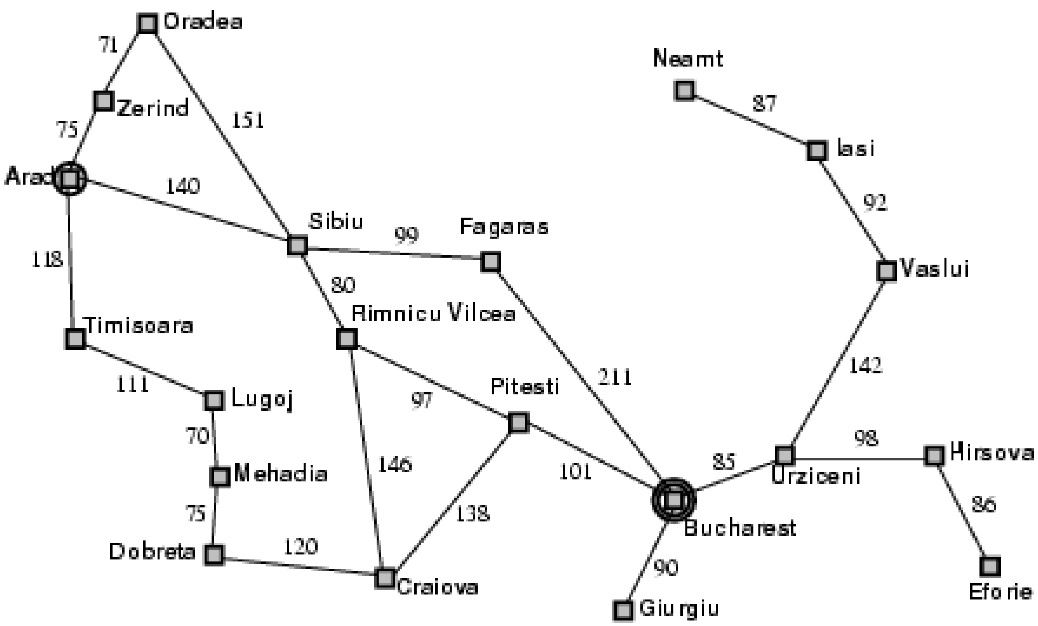

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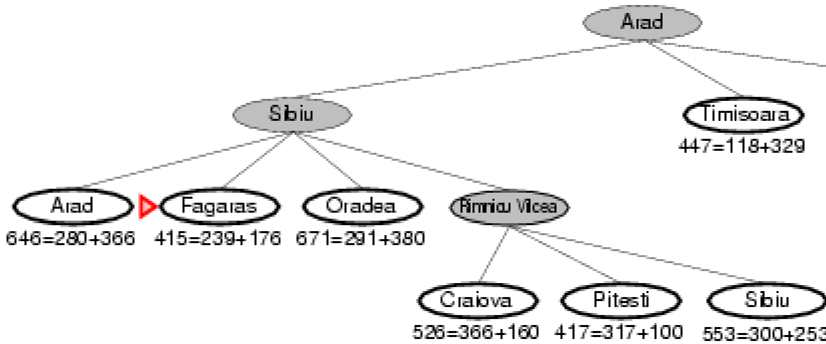

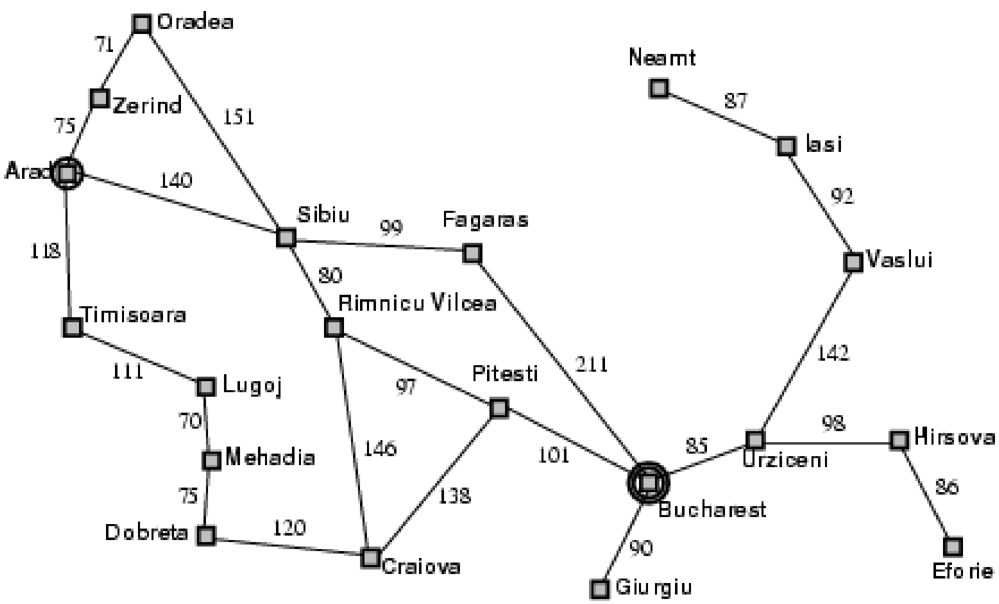

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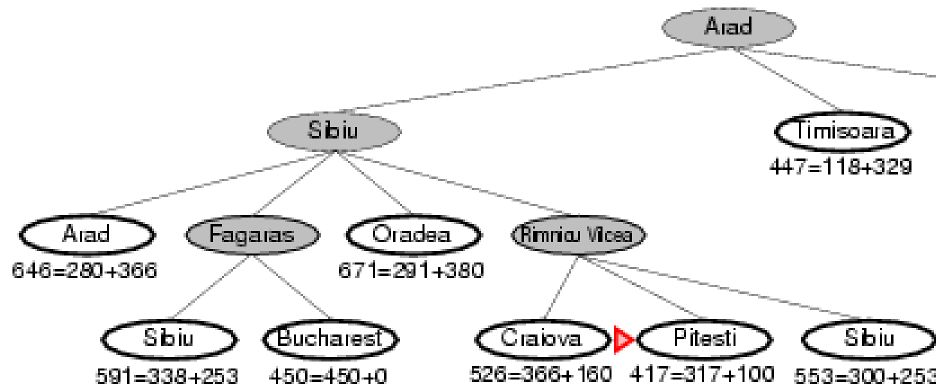

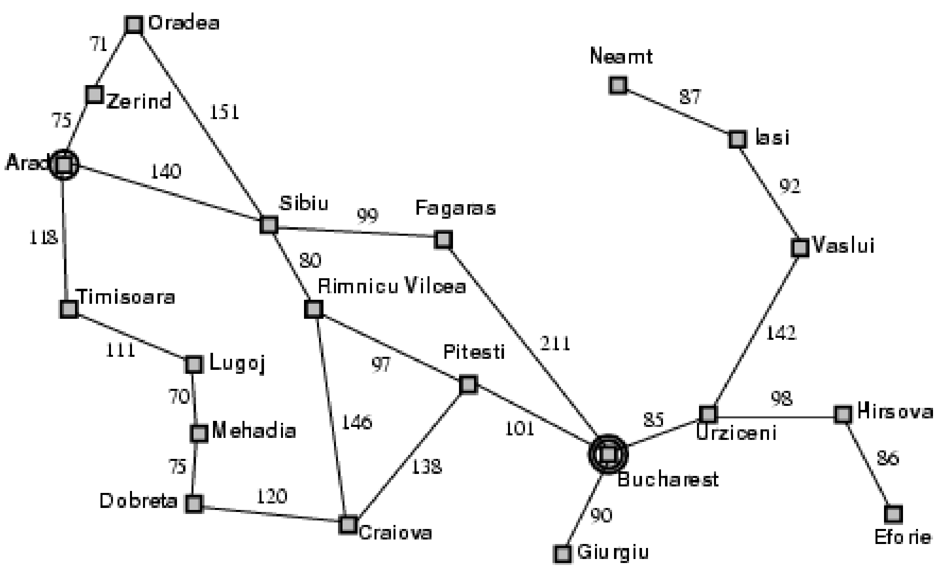

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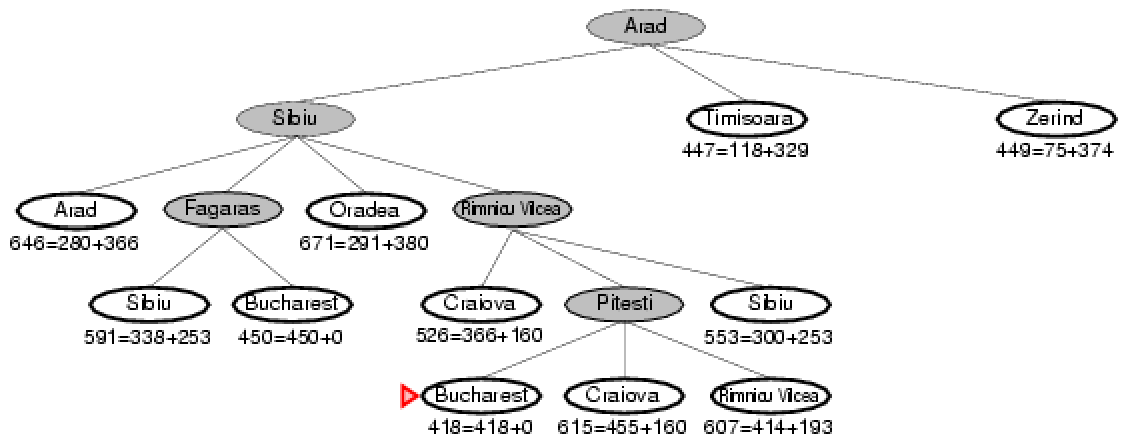

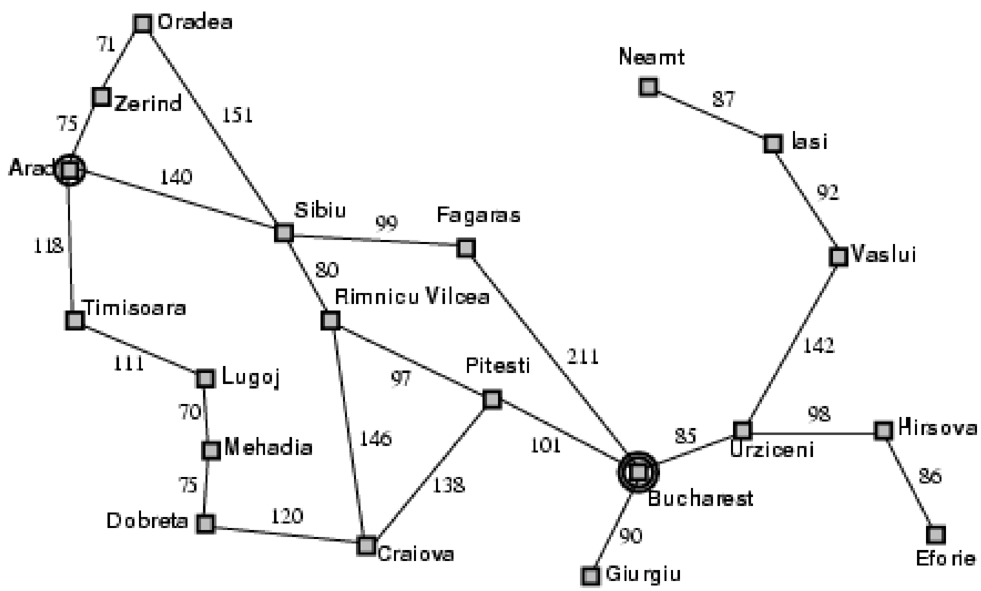

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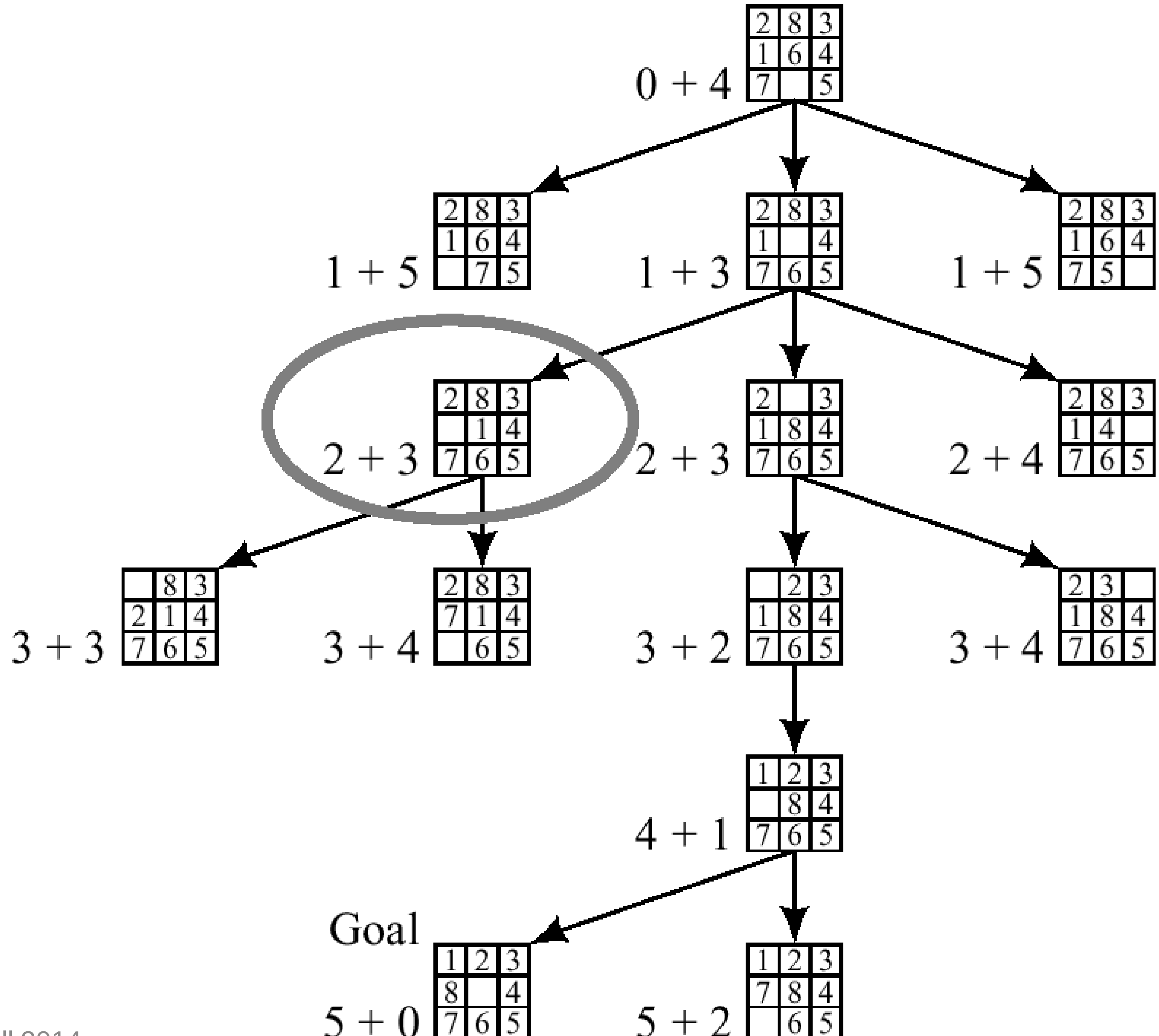
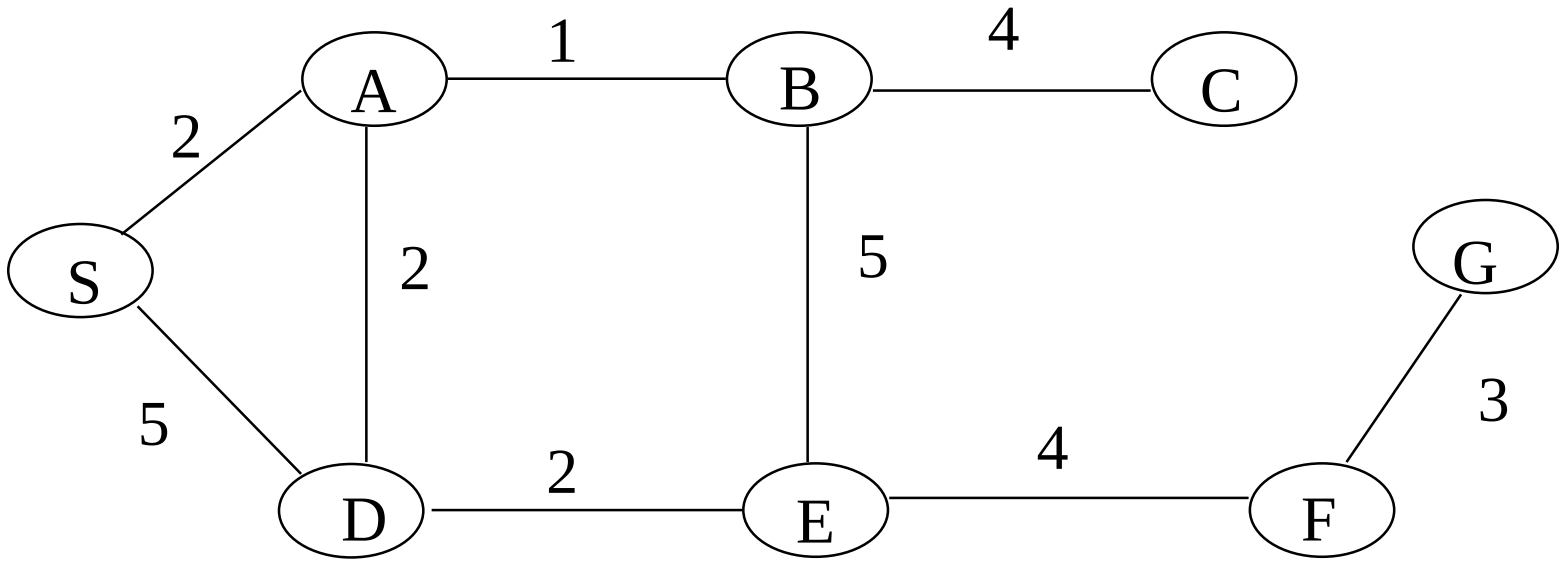
A* on 8-Puzzle with h(n) = # misplaced tiles



A*- a Special Best-First Search

- Goal: find a minimum sum-cost path
- Notation:
  - c(n,n') - cost of arc (n,n')
  - g(n) = cost of current path from start to node n in the search tree.
  - h(n) = estimate of the cheapest cost of a path from n to a goal.
  - evaluation function: f = g+h
- f(n) estimates the cheapest cost solution path that goes through n.
  - h*(n) is the true cheapest cost from n to a goal.
  - g*(n) is the true shortest path from the start s, to n.
  - C* is the cost of optimal solution.

- If the heuristic function, h always underestimates the true cost (h(n) is smaller than h*(n)), then A* is guaranteed to find an optimal solution.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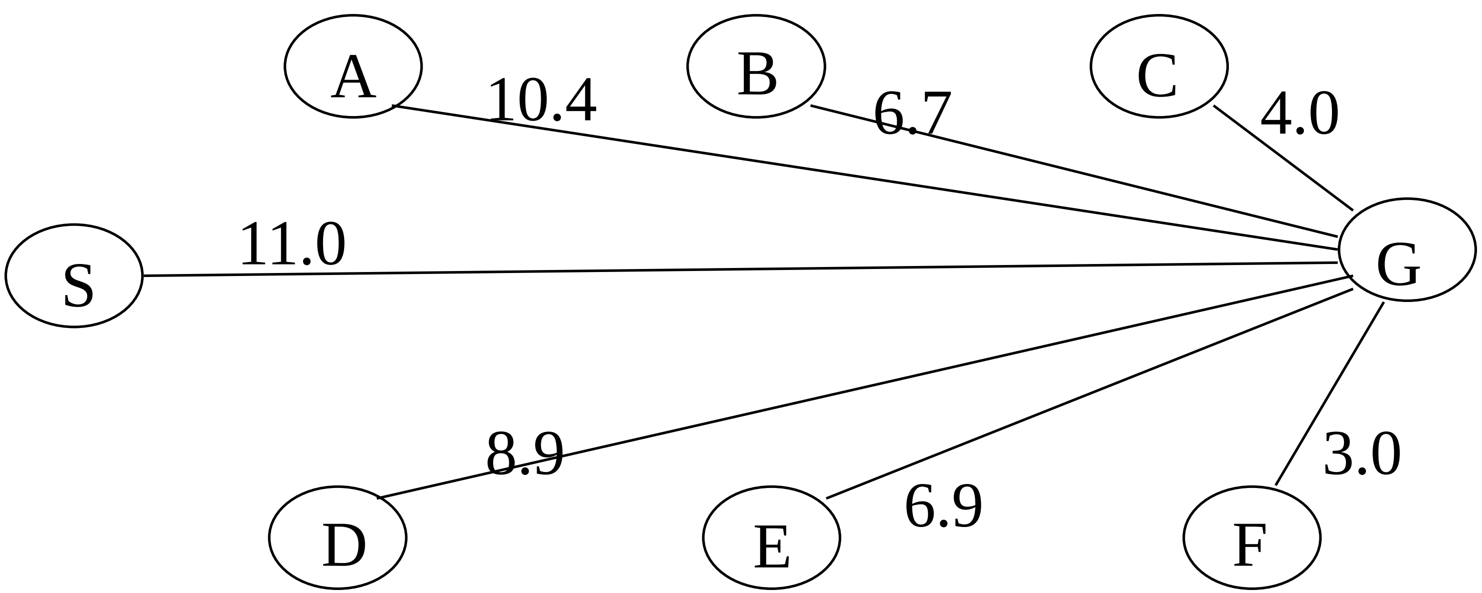
A
B
C
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10.4
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4.0
11.0
8.9
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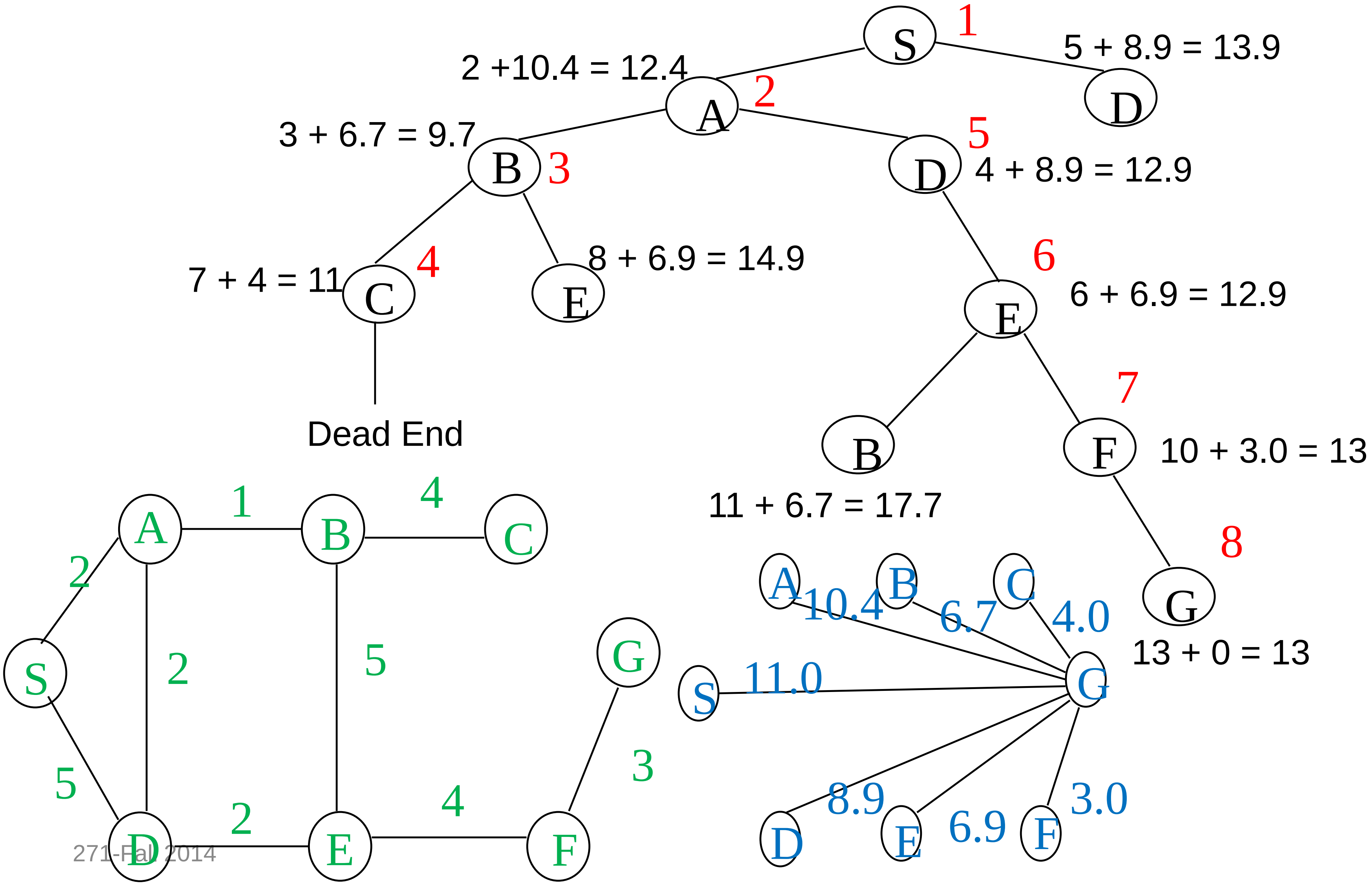
3.0

Example of A* Algorithm in Action



Algorithm A* (with any h on search Graph)

- Input: an implicit search graph problem with cost on the arcs
- Output: the minimal cost path from start node to a goal node.
  - 1. Put the start node s on OPEN.
  - 2. If OPEN is empty, exit with failure
  - 3. Remove from OPEN and place on CLOSED a node n having minimum f.
  - 4. If n is a goal node exit successfully with a solution path obtained by tracing back the pointers from n to s.
  - 5. Otherwise, expand n generating its children and directing pointers from each child node to n.
    - For every child node n' do
      - evaluate h(n') and compute $f(n') = g(n') + h(n') = g(n) + c(n,n') + h(n')$
      - If n' is already on OPEN or CLOSED compare its new f with the old f. If the new value is higher, discard the node. Otherwise, replace old f with new f and reopen the node.
      - Else, put n' with its f value in the right order in OPEN
  - 6. Go to step 2.

Behavior of A - Termination/Completeness

- Theorem (completeness) (Hart, Nilsson and Raphael, 1968)
  - A* always terminates with a solution path (h is not necessarily admissible) if
    - costs on arcs are positive, above epsilon
    - branching degree is finite.
- Proof: The evaluation function f of nodes expanded must increase eventually (since paths are longer and more costly) until all the nodes on a solution path are expanded.

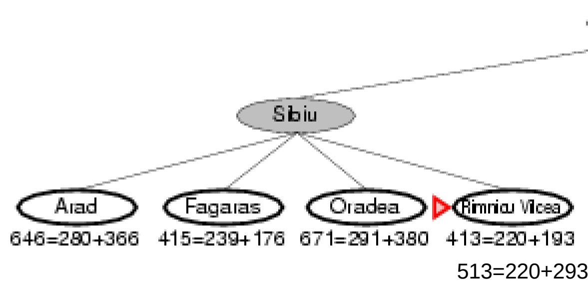
Admissible A*

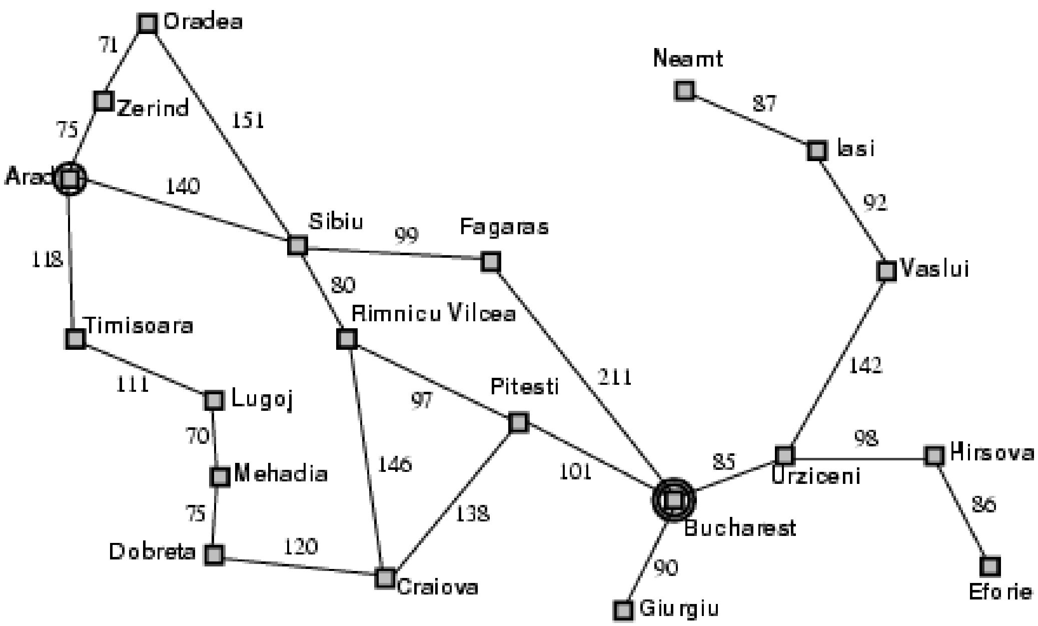
- The heuristic function h(n) is called admissible if h(n) is never larger than h*(n), namely h(n) is always less or equal to true cheapest cost from n to the goal.

- A* is admissible if it uses an admissible heuristic, and h(goal) = 0.

- If the heuristic function, h always underestimates the true cost (h(n) is smaller than h*(n)), then A* is guaranteed to find an optimal solution.

A* with inadmissible h



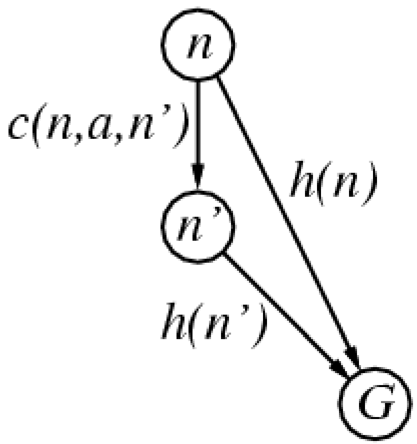


Straight–line distance to Bucharest		
Arad	366	
Bucharest	0	
Craiova	160	
Dobreta	242	
Eforie	161	
Fagaras	176	
Giurgiu	77	
Hirsova	151	
Iasi	226	
Lugoj	244	
Mehadia	241	
Neamt	234	
Oradea	380	
Pitesti	10	
Rimnicu Vilcea	193	→293
Sibiu	253	
Timisoara	329	
Urziceni	80	
Vaslui	199	
Zerind	374	

Consistent (monotone) Heuristics

- A heuristic is consistent if for every node *n*, every successor *n'* of *n* generated by any action *a*,

  $h(n) \leq c(n,a,n') + h(n')$



- If *h* is consistent, we have

$$
\begin{aligned}
f(n') &= g(n') + h(n') \\
&= g(n) + c(n,a,n') + h(n') \\
&\geq g(n) + h(n) \\
&= f(n)
\end{aligned}
$$

- i.e., *f(n)* is non-decreasing along any path.

- Theorem: If *h(n)* is consistent, f along any path is non-decreasing.
- Corollary: the f values seen by A* are non-decreasing.

Consistent Heuristics

- If h is consistent and h(goal)=0 then h is admissible
  - Proof: (by induction of distance from the goal)

- An A* guided by consistent heuristic finds an optimal paths to all expanded nodes, namely $g(n) = g^*(n)$ for any closed n.

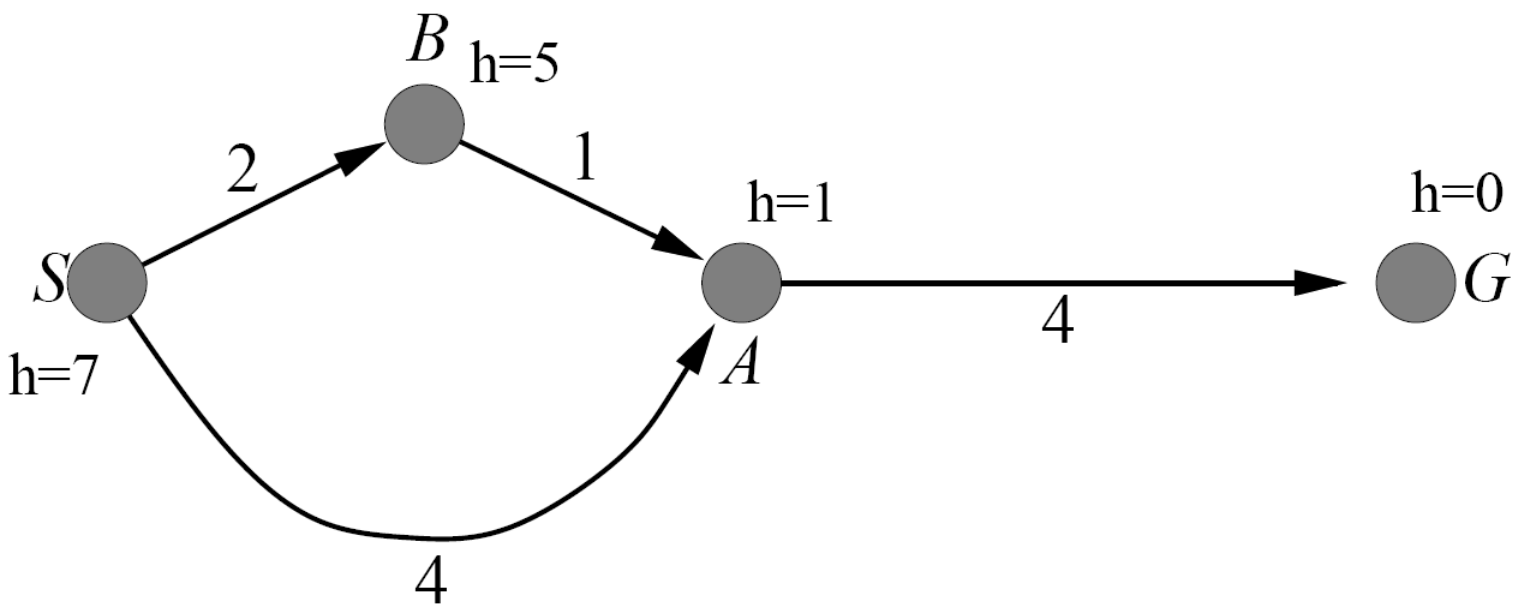
  - Proof: Assume $g(n) > g^*(n)$ and n expanded along a non-optimal path.
  - Let n' be the shallowest OPEN node on optimal path p to n $\rightarrow$
  - $g(n') = g^*(n')$ and therefor $f(n')=g^*(n')+h(n')$
  - Due to consistency we get $f(n') <= g^*(n')+k(n',n)+h(n)$
  - Since $g^*(n) = g^*(n')+k(n',n)$ along the optimal path, we get that
  - $f(n') <= g^*(n) + h(n)$
  - And since $g(n) > g^*(n)$ then $f(n') < g(n)+h(n) = f(n)$, contradiction

Behavior of A* - Optimality

- Theorem (completeness for optimal solution) (HNL, 1968):
  - If the heuristic function is
    - admissible (tree search or graph search with explored node re-opening)
    - Consistent (graph search w/o explored node re-opening)
  - then A* finds an optimal solution.

- Proof:
  - 1. A*(admissible/consistent) will expand only nodes whose f-values are less (or equal) to the optimal cost path C* (f(n) is less-or-equal C*).
  - 2. The evaluation function of a goal node along an optimal path equals C*.
- Lemma:
  - Anytime before A*(admissible/consistent) terminates there exists and OPEN node n’ on an optimal path with f(n’) <= C*.

Inconsistent but admissible



Consistency : $h(n_i) \le c(n_i,n_j) + h(n_j)$

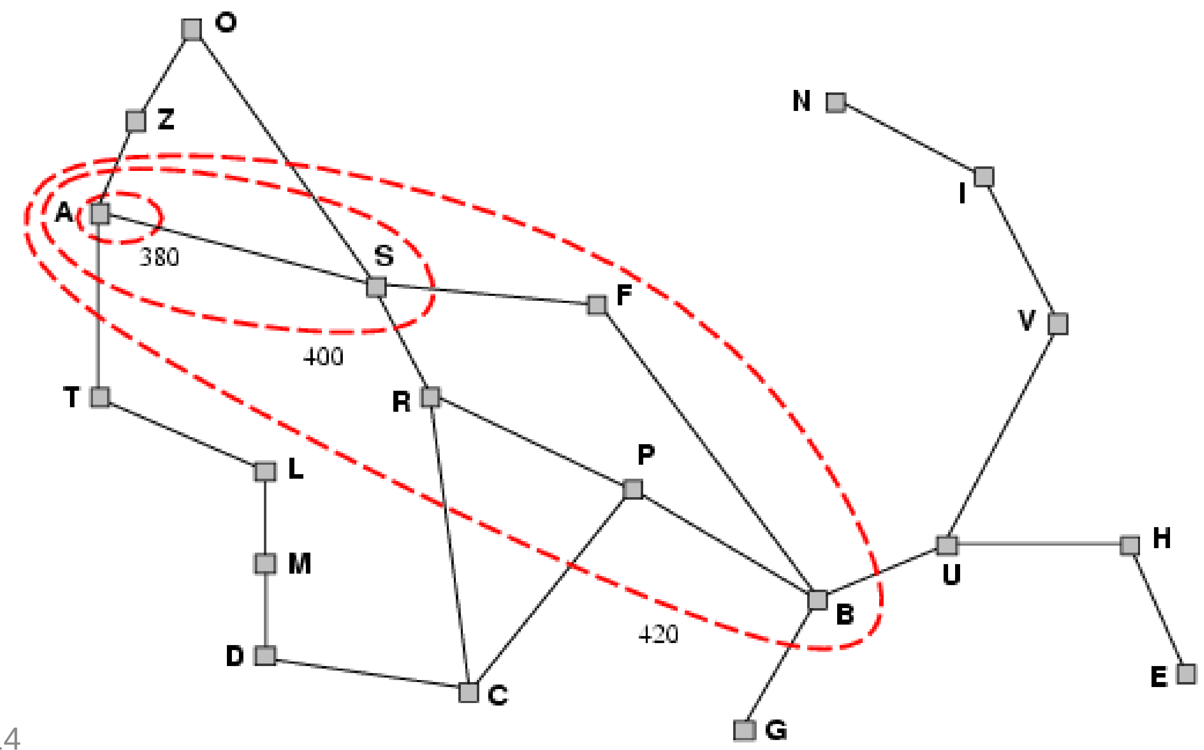
or $c(n_i,n_j) \ge h(n_i) - h(n_j)$

or $c(n_i,n_j) \ge \Delta h$

A^* with Consistent Heuristics

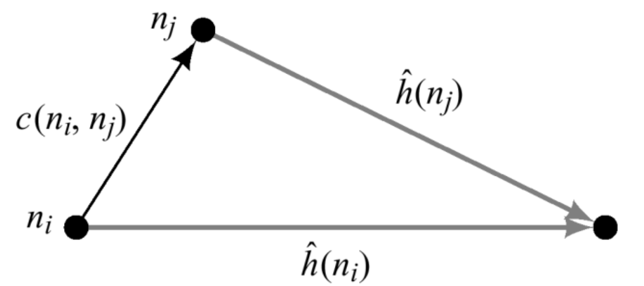
- A^* expands nodes in order of increasing f value

- Gradually adds "f-contours" of nodes
- Contour i has all nodes with $f=f_i$, where $f_i < f_{i+1}$



Summary of Consistent Heuristics

- h is consistent if the heuristic function satisfies triangle inequality for every n and its child node n': $h(n_i) \le h(n_j) + c(n_i,n_j)$



$$\hat{h}(n_i) \le c(n_i, n_j) + \hat{h}(n_j)$$

- When h is consistent, the f values of nodes expanded by A* are never decreasing.
- When A* selected n for expansion it already found the shortest path to it.
- When h is consistent every node is expanded once (no need to check for duplicates).
- Normally the heuristics we encounter are consistent
  – the number of misplaced tiles
  – Manhattan distance
  – straight-line distance

Summary so far

- Best-First Search : f
- A* : f = g + h
- Admissible heuristic : h <= h*
- Consistent heuristic : $h(n_i) <= c(n_i,n_j) + h(n_j)$
- Optimality guaranteed if admissible/consistent

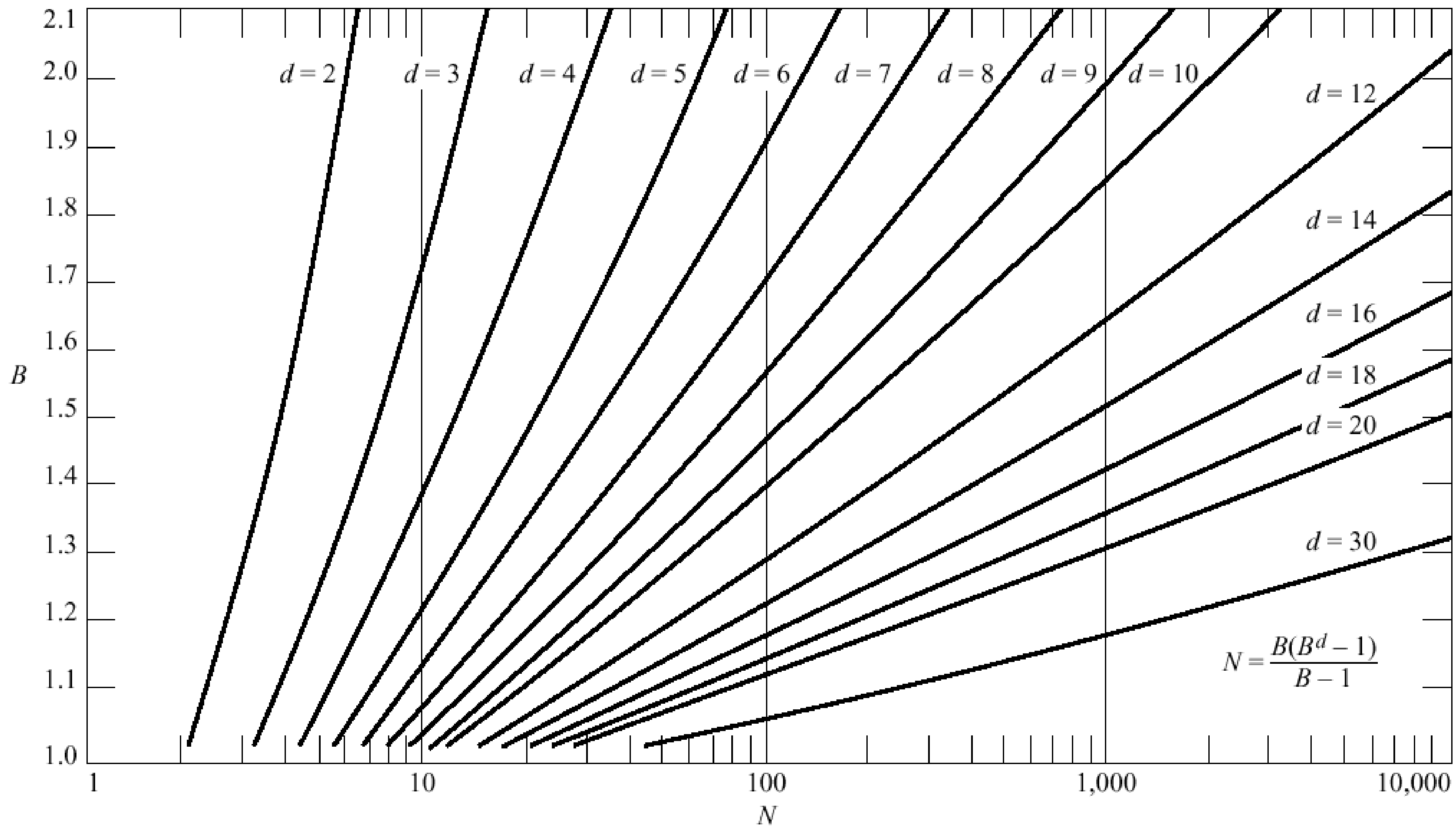
A* properties

- A* expands every path along which $f(n) < C^*$
- A* will never expand any node such that $f(n) > C^*$
- If h is consistent A* will expand any node such that $f(n) < C^*$
- Therefore, A* expands all the nodes for which $f(n) < C^*$ and a subset of the nodes for which $f(n) = C^*$.
- Therefore, if $h_1(n) < h_2(n)$ clearly the subset of nodes expanded by h_2 is smaller.

Complexity of A*

- A* is optimally efficient (Dechter and Pearl 1985):
  - It can be shown that all algorithms that do not expand a node which A* did expand (inside the contours) may miss an optimal solution
- A* worst-case time complexity:
  - is exponential unless the heuristic function is very accurate
- If h is exact ($h = h^*$)
  - search focus only on optimal paths
- Main problem: space complexity is exponential
- Effective branching factor:
  - Number of nodes generated by a “typical” search node
  - Approximately : $b^* = N^{(1/d)}$

The Effective Branching Factor



Properties of A*

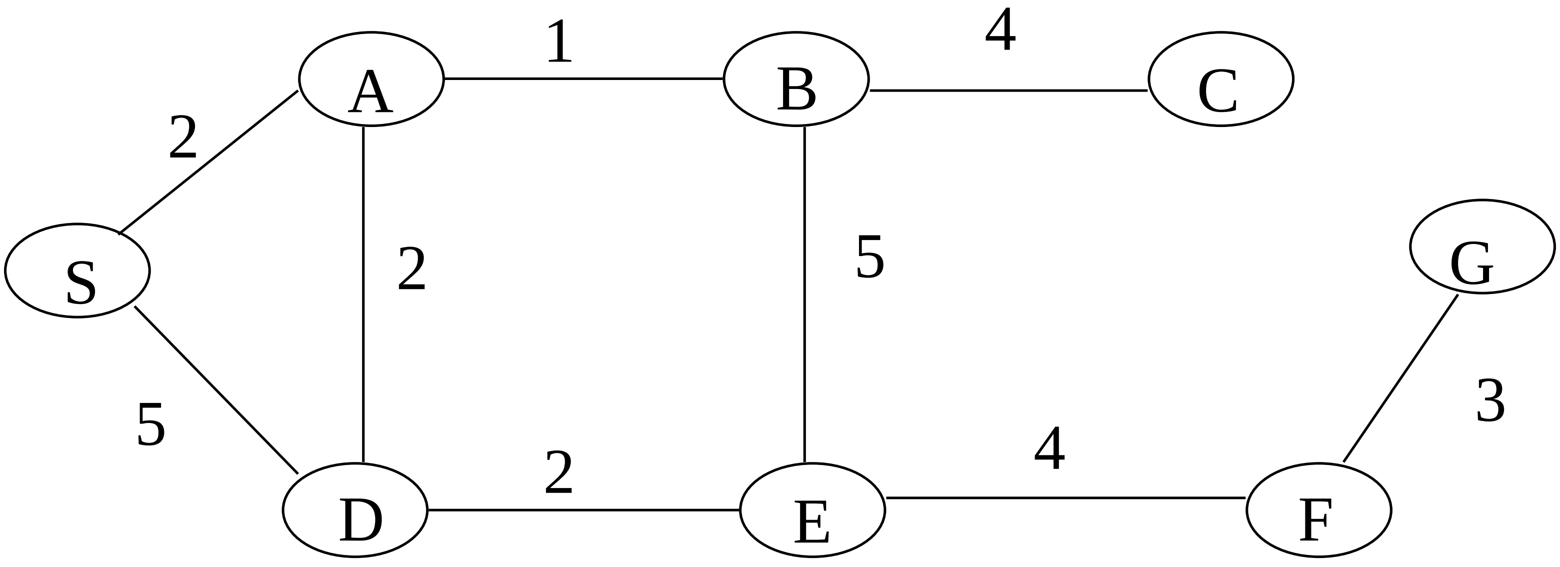
Complete?? Yes, unless there are infinitely many nodes with $f \leq f(G)$

Time?? Exponential in [relative error in h $\times$ length of soln.]

Space?? Keeps all nodes in memory

Optimal?? Yes—cannot expand f_{i+1} until f_i is finished

A* expands all nodes with $f(n) < C^*$
A* expands some nodes with $f(n) = C^*$
A* expands no nodes with $f(n) > C^*$



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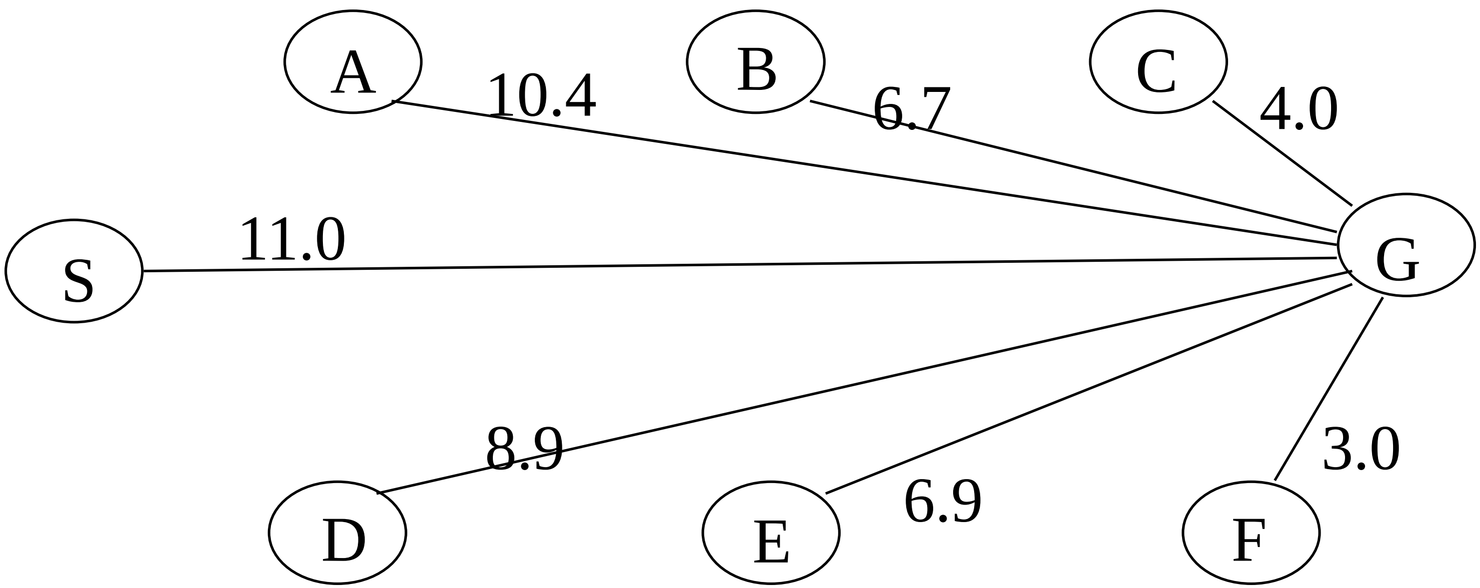

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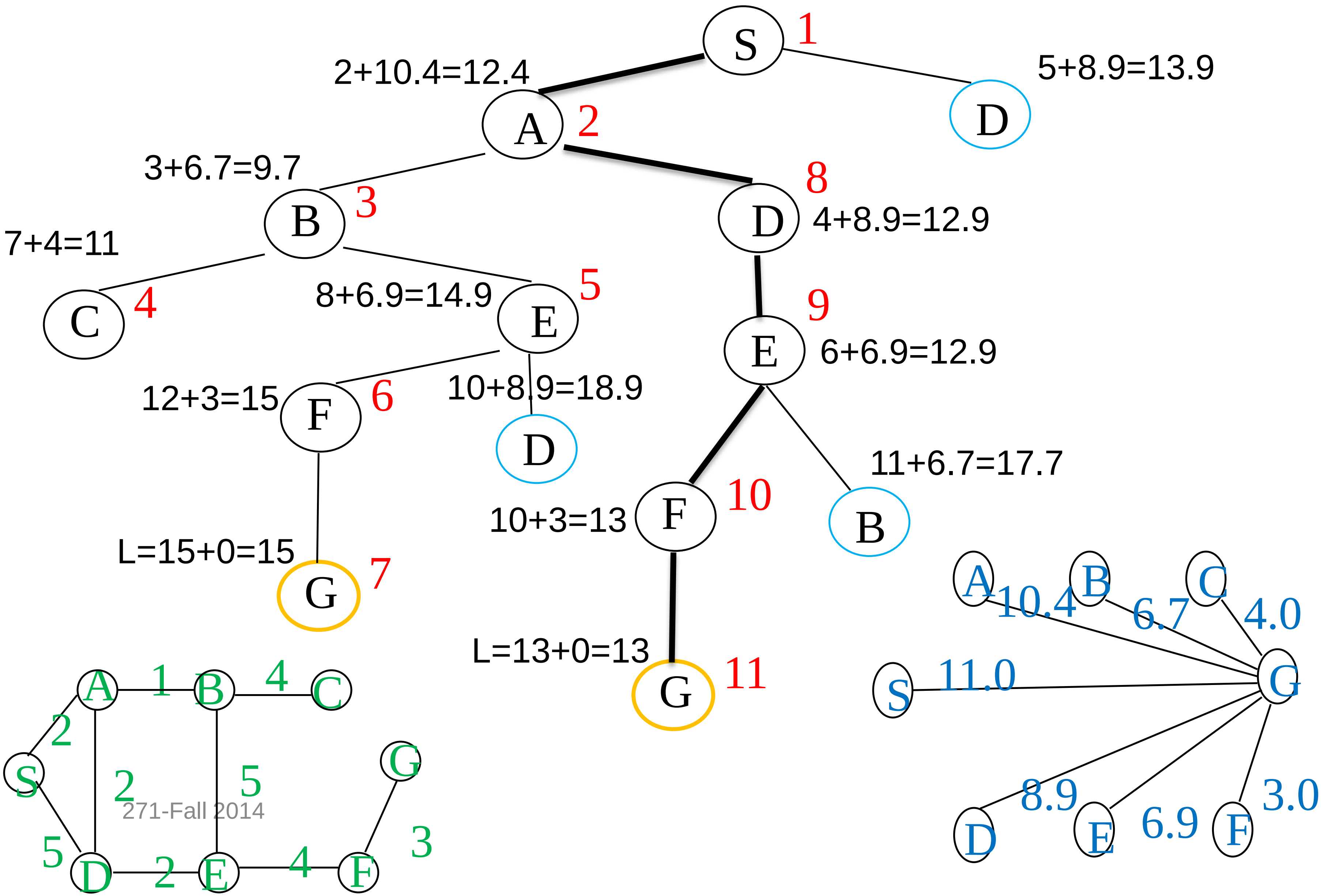
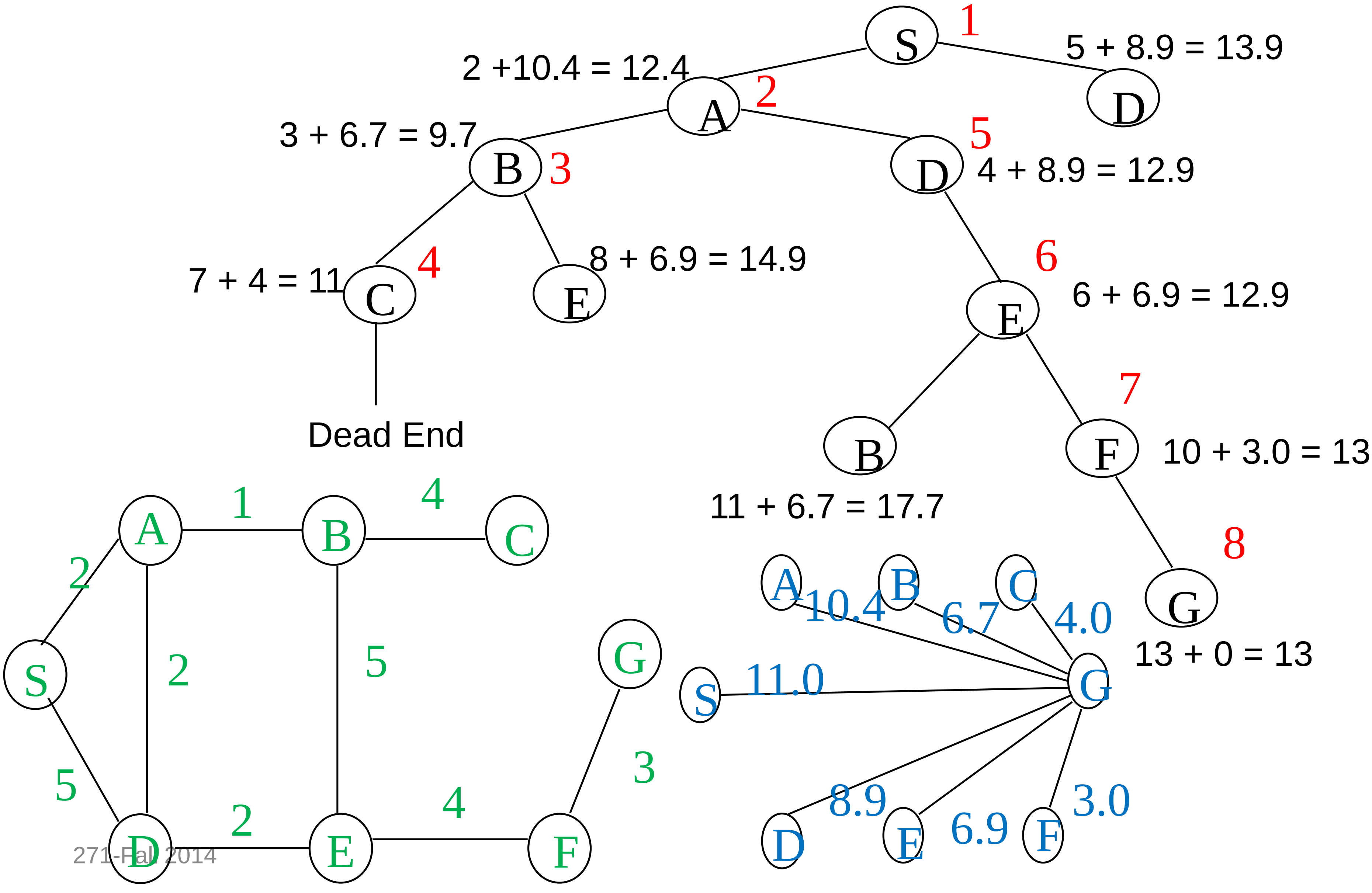
4.0
11.0
8.9
6.9
3.0

Example of Branch and Bound in action



Example of A* Algorithm in Action



Pseudocode for Branch and Bound Search (An informed depth-first search)

```
Initialize: Let Q = {S}, L=∞

While Q is not empty
        pull Q1, the first element in Q
        if f(Q1)>=L, skip it
        if Q1 is a goal compute the cost of the solution and update
                L <-- minimum (new cost, old cost)
        else
                child_nodes = expand(Q1),
               <eliminate child_nodes which represent simple loops>,
                For each child node n do:
                        evaluate f(n).  If f(n) is greater than L discard n.
                end-for
                Put remaining child_nodes on top of queue in the order of their f.
        end
Continue
```

Properties of Branch-and-Bound

- Not guaranteed to terminate unless
  - has depth-bound
  - admissible f and reasonable L
- Optimal:
  - finds an optimal solution (f is admissible)
- Time complexity: exponential
- Space complexity: can be linear
- Advantage:
  - anytime property
- Note : unlike A*, BnB may (will) expand nodes $f>C^*$.

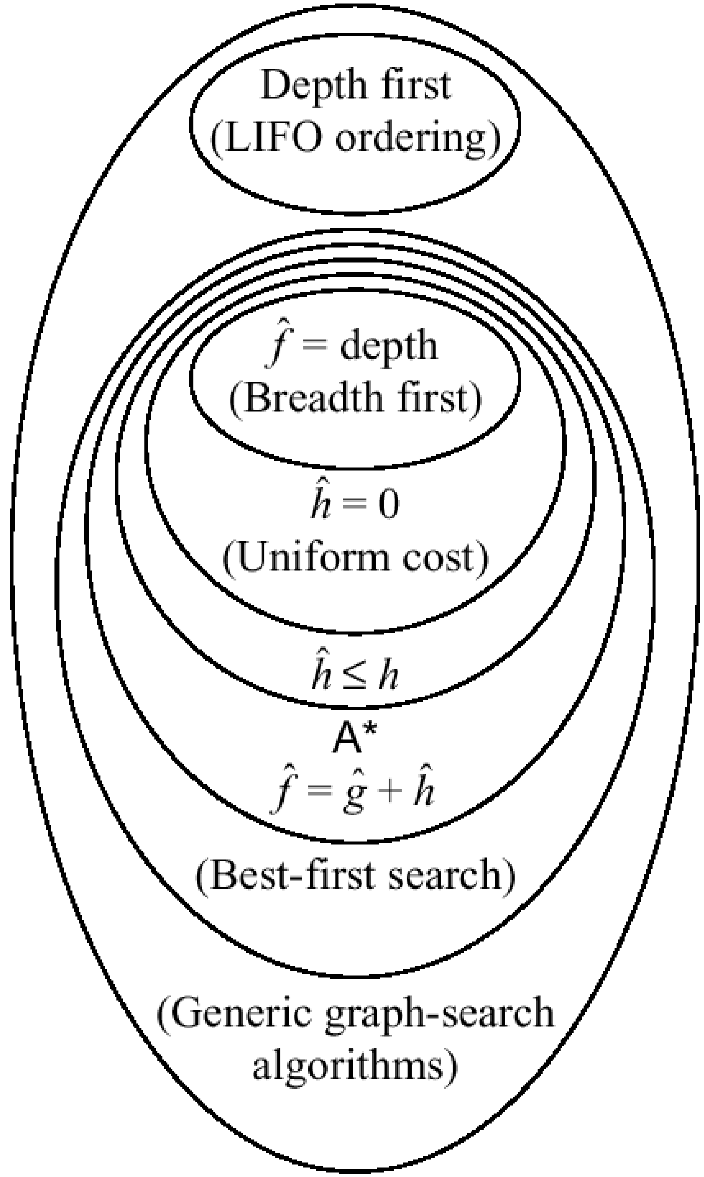
Iterative Deepening A* (IDA*) (combining Branch-and-Bound and A*)

- Initialize: f <-- the evaluation function of the start node
- until goal node is found
  - Loop:
    - Do Branch-and-bound with upper-bound L equal to current evaluation function f.
    - Increment evaluation function to next contour level
  - end

- Properties:
  - Guarantee to find an optimal solution
  - time: exponential, like A*
  - space: linear, like B&B.

  - Problems: The number of iterations may be large.

Relationships among Search Algorithms



Effectiveness of heuristic search

- How quality of heuristic impact search?
- What is the time and space complexity?
- Is any algorithm better? Worse?
- Case study: the 8-puzzle

Admissible and Consistent Heuristics?

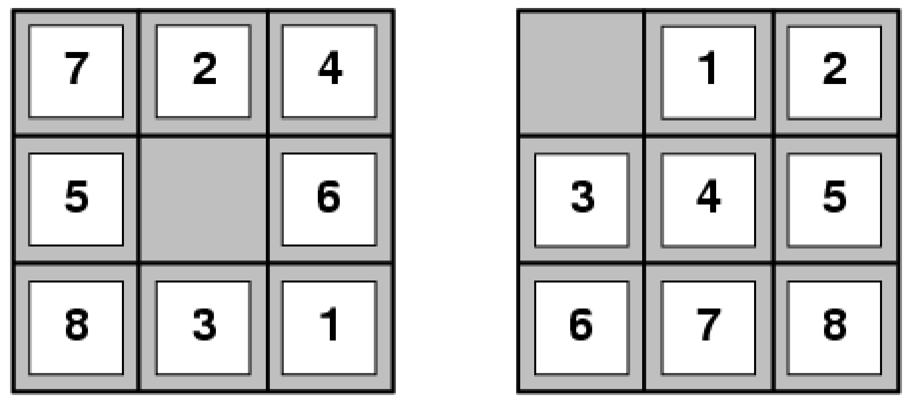
E.g., for the 8-puzzle:

- $h_1(n)$ = number of misplaced tiles
- $h_2(n)$ = total Manhattan distance

(i.e., no. of squares from desired location of each tile)

The true cost is 26.

Average cost for 8-puzzle is 22. Branching degree 3.



Start State Goal State

- $h_1(S)$ = ? 8
- $h_2(S)$ = ? 3+1+2+2+2+3+3+2 = 18

Effectiveness of A* Search Algorithm

Average number of nodes expanded

d	IDS	A*(h1)	A*(h2)
2	10	6	6
4	112	13	12
8	6384	39	25
12	364404	227	73
14	3473941	539	113
20	------------	7276	676
24	------------	39135	1641

Average over 100 randomly generated 8-puzzle problems
h1 = number of tiles in the wrong position
h2 = sum of Manhattan distances

Dominance

- Definition: If $h_2(n) \geq h_1(n)$ for all n (both admissible) then h_2 dominates h_1

- *Is* h_2 better for search?

- Typical search costs (average number of nodes expanded):

- *d=12* IDS = 3,644,035 nodes
  $A^*(h_1)$ = 227 nodes
  $A^*(h_2)$ = 73 nodes
- *d=24* IDS = out of memory
  $A^*(h_1)$ = 39,135 nodes
  $A^*(h_2)$ = 1,641 nodes

Heuristic's Dominance and Pruning Power

- Definition:
  - A heuristic function h_2 (strictly) dominates h_1 if both are admissible and for every node n, $h_2(n)$ is (strictly) greater than $h_1(n)$.
- Theorem (Hart, Nilsson and Raphale, 1968):
  - An A* search with a dominating heuristic function h_2 has the property that any node it expands is also expanded by A* with h_1.
- Question: Does Manhattan distance dominate the number of misplaced tiles?
- Extreme cases
  - $h = 0$
  - $h = h^*$

Inventing Heuristics automatically

- Examples of Heuristic Functions for A*
  - The 8-puzzle problem
    - The number of tiles in the wrong position
      - is this admissible?
    - Manhattan distance
      - is this admissible?
  - How can we invent admissible heuristics in general?
    - look at “relaxed” problem where constraints are removed
      - e.g.., we can move in straight lines between cities
      - e.g.., we can move tiles independently of each other

Inventing Heuristics Automatically (cont.)

- How did we
  - find h1 and h2 for the 8-puzzle?
  - verify admissibility?
  - prove that straight-line distance is admissible? MST admissible?
- Hypothetical answer:
  - Heuristic are generated from relaxed problems
  - Hypothesis: relaxed problems are easier to solve
- In relaxed models the search space has more operators or more directed arcs
- Example: 8 puzzle:
  - Rule : a tile can be moved from A to B, iff
    - A and B are adjacent
    - B is blank
  - We can generate relaxed problems by removing one or more of the conditions
    - … A and B are adjacent & B is blank
    - ... if B is blank

Relaxed Problems

- A problem with fewer restrictions on the actions is called a relaxed problem

- The cost of an optimal solution to a relaxed problem is an admissible heuristic for the original problem

- If the rules of the 8-puzzle are relaxed so that a tile can move anywhere, then *$h_1(n)$ (number of misplaced tiles)* gives the shortest solution

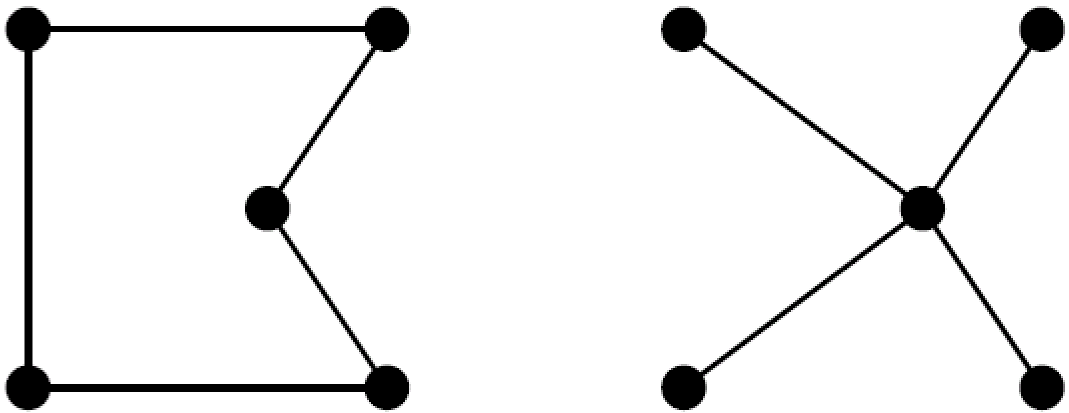
- If the rules are relaxed so that a tile can move to any h/v adjacent square, then *$h_2(n)$ (Manhatten distance)* gives the shortest solution

Generating heuristics (cont.)

- Example: TSP
- Find a tour. A tour is:
  - 1. A graph with subset of edges
  - 2. Connected
  - 3. Total length of edges minimized
  - 4. Each node has degree 2
- Eliminating 4 yields MST.

Relaxed problems contd.

Well-known example: travelling salesperson problem (TSP)
Find the shortest tour visiting all cities exactly once



Minimum spanning tree can be computed in $O(n^2)$
and is a lower bound on the shortest (open) tour

Automating Heuristic generation

- Use STRIPs language representation:
- Operators:
  - pre-conditions, add-list, delete list
- 8-puzzle example:
  - on(x,y), clear(y) adj(y,z) ,tiles x1,…,x8
- States: conjunction of predicates:
  - on(x1,c1),on(x2,c2)….on(x8,c8),clear(c9)
- move(x,c1,c2) (move tile x from location c1 to location c2)
  - pre-cond: on(x1,c1), clear(c2), adj(c1,c2)
  - add-list: on(x1,c2), clear(c1)
  - delete-list: on(x1,c1), clear(c2)
- Relaxation:
  - Remove from precondition: clear(c2), adj(c2,c3) → #misplaced tiles
  - Remove clear(c2) → Manhattan distance
  - Remove adj(c2,c3) → h3, a new procedure that transfers to the empty location a tile appearing there in the goal
- The space of relaxations can be enriched by predicate refinements
  - adj(y,z) = iff neigbour(y,z) and same-line(y,z)

Heuristic generation

- Theorem: Heuristics that are generated from relaxed models are consistent.

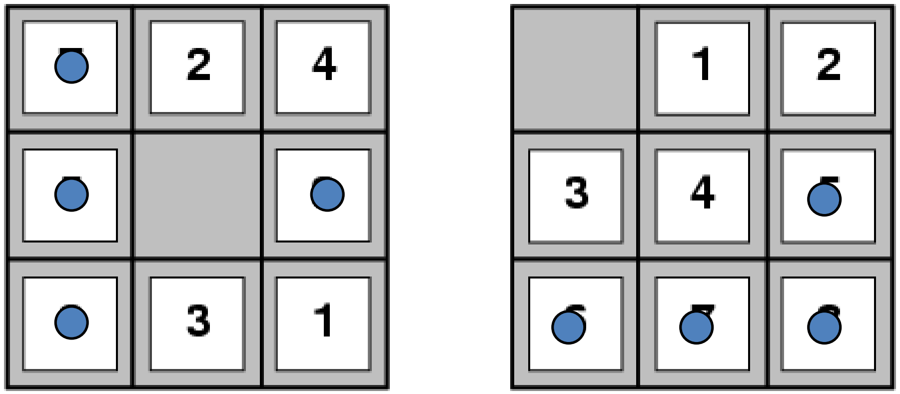
- Proof: h is true shortest path in a relaxed model
  - $h(n) <= c'(n,n')+h(n')$ (c' are shortest distances in relaxed graph)
  - $c'(n,n') <= c(n,n')$
  - $\rightarrow h(n) <= c(n,n')+h(n')$

Heuristic generation

- Total (time) complexity = heuristic computation + nodes expanded
- More powerful heuristic – harder to compute, but more pruning power (fewer nodes expanded)
- Problem:
  - not every relaxed problem is easy
    - How to recognize a relaxed easy problem
    - A proposal: a problem is easy if it can be solved optimally by a greedy algorithm
- Q: what if neither h_1 nor h_2 is clearly better? $\max(h_1, h_2)$
- Often, a simpler problem which is more constrained is easier; will provide a good upper-bound.

Improving Heuristics

- Reinforcement learning.
- Pattern Databases: you can solve optimally a sub-problem



Start State Goal State

Pattern Databases

- For sliding tiles and Rubic’s cube
- For a subset of the tiles compute shortest path to the goal using breadth-first search
- For 15 puzzles, if we have 7 fringe tiles and one blank, the number of patterns to store are 16!/(16-8)! = 518,918,400.
- For each table entry we store the shortest number of moves to the goal from the current location.
- Use different subsets of tiles and take the max heuristic during IDA* search. The number of nodes to solve 15 puzzles was reduced by a factor of 346 (Culberson and Schaeffer)
- How can this be generalized? (a possible project)

Beyond Classical Search

- AND/OR search spaces
  - Decomposable independent problems
  - Searching with non-deterministic actions (erratic vacuum)
  - Using AND/OR search spaces; solution is a contingent plan
- Local search for optimization
  - Greedy hill-climbing search, simulated annealing, local beam search, genetic algorithms.
  - Local search in continuous spaces
  - SLS : "Like climbing Everest in thick fog with amnesia"
- Searching with partial observations
  - Using belief states
- Online search agents and unknown environments
  - Actions, costs, goal-tests are revealed in state only
  - Exploration problems. Safely explorable

Problem-reduction representations AND/OR search spaces

- Decomposable production systems (language parsing)

  Initial database: (C,B,Z)

  Rules: R1: C → (D,L)

  R2: C → (B,M)

  R3: B → (M,M)

  R4: Z → (B,B,M)

  Find a path generating a string with M's only.
- Graphical models
- The tower of Hanoi

  To move n disks from peg 1 to peg 3 using peg 2

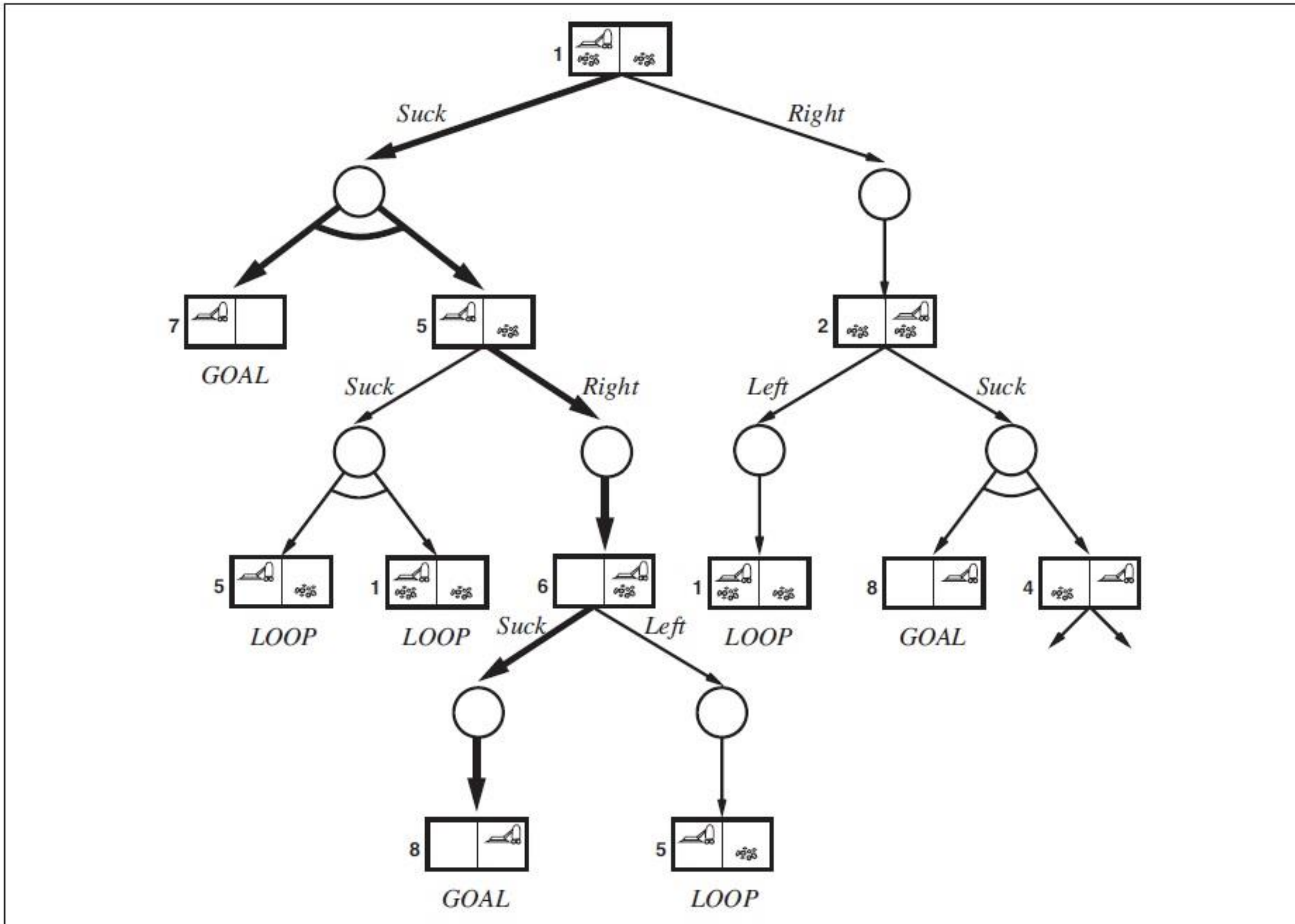
  Move n-1 pegs to peg 2 via peg 3,

  move the nth disk to peg 3,

  move n-1 disks from peg 2 to peg 3 via peg 1.

AND/OR search spaces

non-deterministic actions : the erratic vacuum world



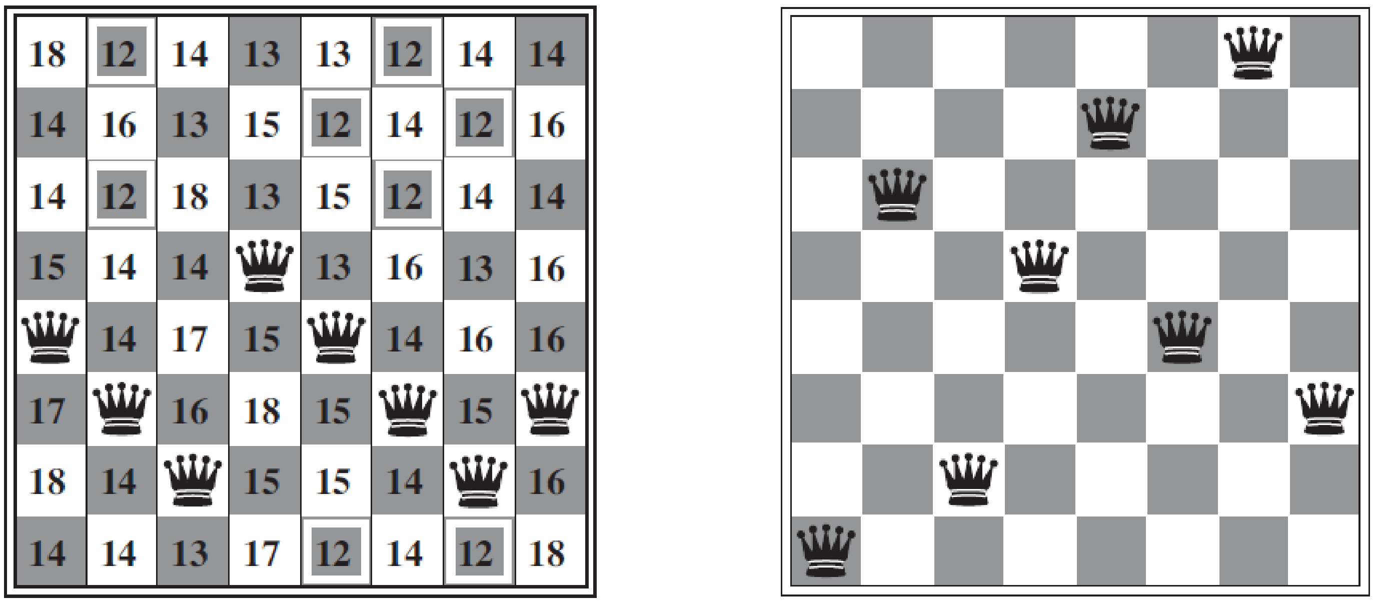
AND/OR Graphs

- Nodes represent subproblems
  - AND links represent subproblem decompositions
  - OR links represent alternative solutions
  - Start node is initial problem
  - Terminal nodes are solved subproblems
- Solution graph
  - It is an AND/OR subgraph such that:
    - It contains the start node
    - All its terminal nodes (nodes with no successors) are solved primitive problems
    - If it contains an AND node A, it must contain the entire group of AND links that leads to children of A.

Algorithms searching AND/OR graphs

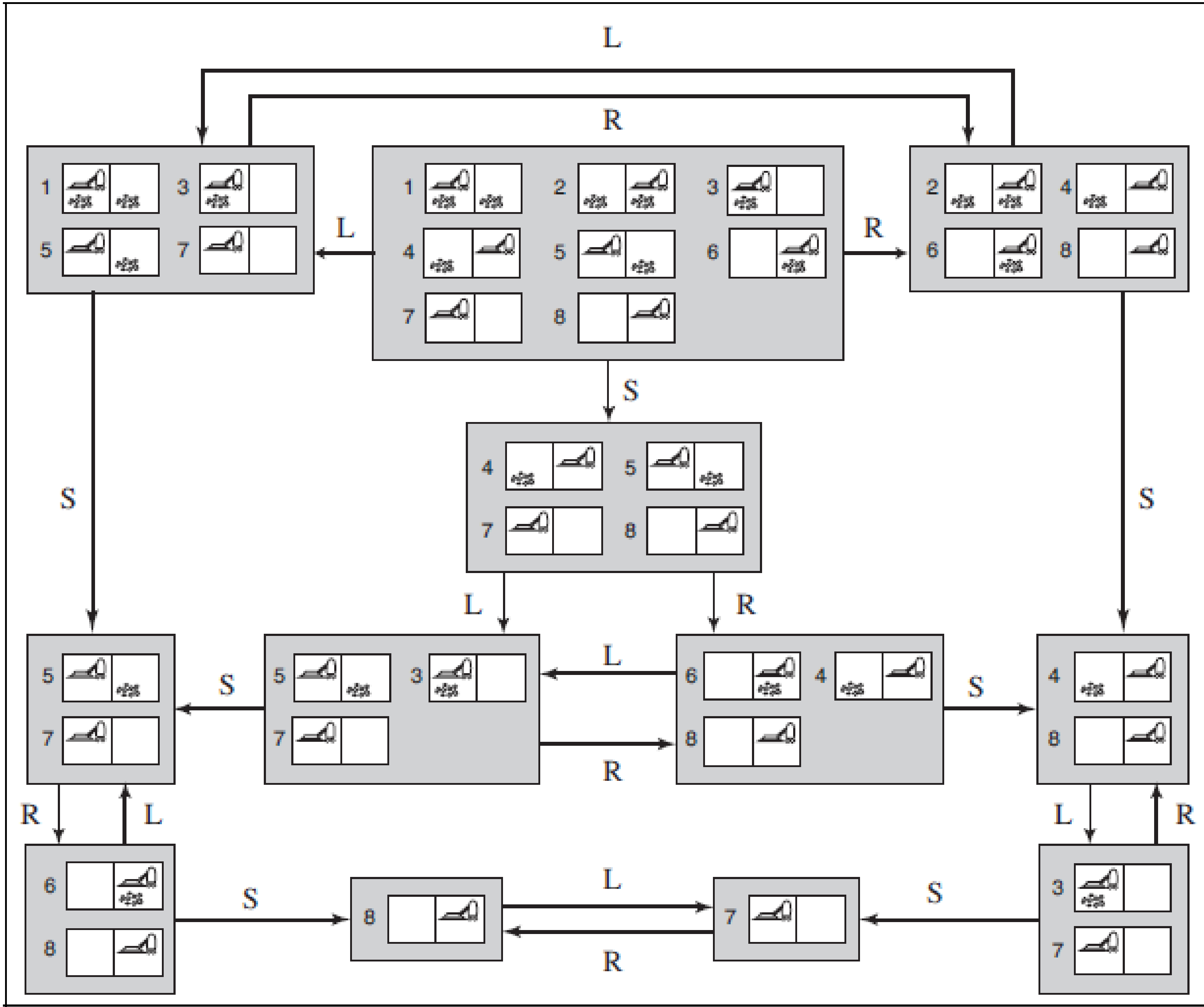
- All algorithms generalize using hyper-arc successors rather than simple arcs.
- AO*: is A* that searches AND/OR graphs for a solution subgraph.
- The cost of a solution graph is the sum cost of it arcs. It can be defined recursively as: $k(n,N) = c\_n+k(n1,N)+\ldots k(n\_k,N)$
- h*(n) is the cost of an optimal solution graph from n to a set of goal nodes
- h(n) is an admissible heuristic for h*(n)
- Monotonicity:
- $h(n) <= c+h(n1)+\ldots h(nk)$ where n1,…nk are successors of n
- AO* is guaranteed to find an optimal solution when it terminates if the heuristic function is admissible

Local Search



(a) (b)

Figure 4.3 (a) An 8-queens state with heuristic cost estimate $h = 17$, showing the value of h for each possible successor obtained by moving a queen within its column. The best moves are marked. (b) A local minimum in the 8-queens state space; the state has $h = 1$ but every successor has a higher cost.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Summary

- In practice we often want the goal with the minimum cost path
- Exhaustive search is impractical except on small problems
- Heuristic estimates of the path cost from a node to the goal can be efficient in reducing the search space.
- The A* algorithm combines all of these ideas with admissible heuristics (which underestimate) , guaranteeing optimality.
- Properties of heuristics:
  - admissibility, consistency, dominance, accuracy
- Reading
  - R&N Chapters 3-4